A COLLECTIVE-GOODS MODEL OF RETAILING

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Introduction

Traditional, monopolistically competitive, theories of retailing, at least those containing unambiguous welfare implications, assume that a firm's overhead, "selling" services are duplicative rather than distinctive and naturally monopolistic. This fairly unrealistic assumption is, for example, crucial to obtaining the most widely discussed result of monopolistic-competition theory, Chamberlain's too-many-firms conclusion.

The standard theories also assume, more realistically, that all of a given retailer's customers pay an identical, constant-per-unit price to that retailer.

It is easy to see, however, that rational price-setting sellers of a duplicative overhead service would first charge lump-sum, "cover" charges reflecting the easily estimated per-customer cost of switching to rival retailers, and then sell their private goods at marginal production costs. Rather than deriving the rational, multi-part, price system suitable to the underlying economic environment, authors in the field of monopolistic-competition have simply imposed single-price systems on their models. These authors have thus overlooked the fact that their empirically justifiable assumption of uniform mark-ups contradicts their more questionable assumption that their monopolistically competing retailers are providing duplicative overhead services. That is, retailers charging the same, constant-per-unit, above-marginal-cost, price to all customers should be assumed to be providing substantially distinct, collective-good, type of overhead services.

Under this alternative assumption on the nature of the sellers' overhead services, uniform mark-ups can be easily rationalized in that they furnish
collective-good suppliers with exactly what such suppliers want — a powerful method of preference-based, personal price discrimination. For such systems force high-level collective-good-demanders — who are also relatively high-probability purchasers of private-good complements — to make expected payments to the retailers that are many times higher than low-level collective-good-demanders.

Admitting distinctive, or non-duplicative, overhead services both eliminates Chamberlain's too-many-firms argument and, under realistically imperfect price discrimination, leads to the conclusion that laissez faire results in too few firms because of each firm's inability to collect for its unique, collective-good, type of overhead service. Consequently, subsidizing the private-good quantities sold by these retailers, rather than exacerbating a too-many-firms problem, moves the equilibrium number of firms towards efficiency.

Section IV of this paper points out that real world policies toward retailers typically do offer an implicit subsidy to the private-good outputs of retailers. As the equilibrium number of firms under this policy is, we shall argue, roughly optimal, we shall simply assume from the start that the number of retail firms is fixed at an approximately optimal level and evaluate the Pareto optimality of the equilibrium outputs of both private and collective goods provided by the fixed number of retailers.

Correspondingly, Sections I-III of the paper will develop a new, distinct-overhead-service, or collective-goods, model of retailing. Besides indicating that there is a laissez-faire underproduction of private-good complements to the distinct collective-good outputs of the various retailers, it will also show that there is an unambiguous overproduction of any given firm's collective-good service once the optimal private-good subsidies are in
place. These results will serve to qualitatively rationalize the entire set of observed policies towards real-world retailers: The private-goods-underproduction result rationalizes observed, implicit, per-unit subsidies to the private-good complements sold by ordinary retailers while the more surprising, collective-goods-overproduction result rationalizes existing laws restricting the appropriability of relatively high-quality retailers, laws restricting resale price maintenance and related vertical arrangements. Although theoretical models have already been developed to rationalize several existing laws restricting the appropriability of sellers of other kinds of collective-goods (Thompson, 1968, 1984), these models: (a) cannot be realistically applied to retailing, and (b) imply a laissez faire underproduction of collective-good as well as private-good complements, thereby calling for significant subsidies to the former complements.

I. The Physical Environment

We consider a world composed of: (1) a fixed number, $R$, of retailers, each supplying a distinct, variable quality, $Q_i$, $i = 1, \ldots, R$, of overhead services to its variable number of customers, $N_i$, and (2) a numeraire private-good output, $Z$, which is unrelated in both consumption and production to the retailers' $Q_i$ outputs. A social optimum, $\{Q^o_i, N^o_i, Z^o\}$, is determined by maximizing a collective utility function:

$$U(Q_1 \cdot f_1(N_1), \ldots, Q_R \cdot f_R(N_R)) + A \cdot Z;$$

where $A$ is a positive constant, the first derivatives, $U_i$, are uniformly positive while the second derivatives, $U_{ii}$, are uniformly negative, and $N^o_i > 1$; subject to the social transformation constraint,
$C_1(Q_1) + \ldots + C_R(Q_R) + Z = 0$, where $C'_i > 0$ and $C''_i > 0$ for all $Q_i$.\(^1\)

The $f_i(N_i)$-functions express the simultaneous-user characteristic of the overhead services, where $f_i(0) = 0$, $f_i(1) = 1$, and, beyond that, $f_i(N_i)$ changes with unit expansions in $N_i$ by amounts equal to the fraction of the highest-use-value-consumer's MRS represented by the additional consumer's MRS, with consumers of $Q_i$ being added in descending order of their MRS's so that the function's first difference from $N_i-1$ to $N_i$, denoted $f'_i(N_i)$, decreases as $N_i$ expands. Thus, $\frac{U_i}{A}$ represents the MRS of the highest-use-value consumer, $\frac{U_i}{A} \cdot f'(2)/A$ represents the MRS of the next-highest-use-value consumer, etc. The pure-collective-good property of the retailers' outputs is apparent from the fact that all consumers of $Q_i$ gain (or lose) simultaneously from expansions in $Q_i$, there being no technology with which to redistribute a given level of $Q_i$ among the members of a given set of $N_i$ users.

Consumers implicitly bear the resource cost of obtaining any given retailer's overhead service. The presence of these utilization costs means that the social optimum generally has $N_i^O < N$, the total population. Assuming that this is always the case and, for notational simplicity, that there is, for each collective good, a consumer with a zero MRS in the optimum, the first-order marginal conditions for a solution to the above-described social optimization problem are:

1. $f'_i(N_i^O) = 0$ and
2. $\frac{U_i f_i(N_i^O)}{A} = C'_i$ for all $i$.

\(^1\)The reason for the linearity-on-Z assumptions is that our subsequent model of market equilibrium will not make standard, Walras-Lindahl, price-taking assumptions and therefore will require conventional, Marshallian, small-industry assumptions to insure that our retailers do not influence the real prices or costs of other retailers merely by inducing alterations in the values or costs of producing the numeraire.
We assume that there is only one such solution for a given distribution of \( Z \) between the consumers and thus a given collective utility function. The first condition means that consumers are added to the \( i^{th} \) retailer until an additional consumer would have a zero real value for the \( i^{th} \) retailer's overhead service. The second is the familiar Lindahl–Samuelson condition that the sum of the individual marginal real values for \( Q_i \) to the existing \( N_i \) users is equal to the real marginal cost of \( Q_i \).

We assume that each retailer is always physically able to raise all of his discriminatory flat-rate prices so as to fully capture the additional real value of any improvement in his overhead services to his existing customers. When, and only when, retailers also know each of the solution demand-prices, price discrimination will be perfect. We shall evaluate the allocational consequences of both perfect and imperfect price discrimination in the analysis that follows.

We assume throughout that our \( R \) overhead services are so perishable, or the numbers of customers so large, that it pays suppliers to observe neither the effective price-offers of other suppliers nor the quantities purchased of their customers from these suppliers. The resulting, simultaneous-marketing, Bertrand interaction contrasts sharply with the sequential marketing interaction described in our earlier analyses (Thompson 1968, 1984). Nevertheless, the interaction will be shown to generate an analogous tendency toward collective-good overproduction by realistically price-discriminating, collective-good suppliers. Furthermore, an even less qualified, unambiguous, overproduction result will emerge for our Bertrand suppliers once their sales of private-good complements are subsidized so as to remove the familiar over-exclusion effects appearing under imperfectly informed selling. Subsidizing the sales of private-good complements of any Bertrand-type collective-good
supplier so as to remove such effects qualifies him as a “retailer”, according to our now-final, theoretical definition of the term. Before considering the realistic case containing imperfectly informed sellers and private-good complements, we consider the highly unrealistic, but theoretically interesting, case of perfect price discrimination.

II. The Impossibility of a Socially Optimal Equilibrium Under Perfect Price Discrimination

We first consider the unjustifiable and unrealistic case in which retailers have complete a priori knowledge of consumer preferences and retailer costs and therefore perfectly price discriminate. Consumers therefore surrender essentially all of their economic surplus from the product of a given retailer to that retailer. The \( i \)th retailer will obviously find it profitable to add customers until \( f'(N_i) = 0 \), trivially satisfying the first social optimality condition in the hypothetical, perfectly discriminatory solution. However, in attempting to have retailers satisfy the second optimality condition, the condition describing the optimal \( Q_i \)-choices, a basic logical problem inevitably appears, one precluding any Bertrand-type, pure-strategy solution to the problem. We develop the point by constructing a sequence of graphs. To start, we construct a graph (Fig. 1) depicting the initial indifference curve of the highest-demand-price customer of retailer \( i \) when he buys none of the outputs of the other retailers and has a numeraire endowment of \( z_0 \). The consumer’s utility level is therefore given by \( V(0,0) + az_0 = V(Q_1,0) + az \) where the vector zero term, \( 0 \), represents the initial values of the vector, \( Q_{-1} = Q_1, \ldots, Q_{i-1}, Q_{i+1}, \ldots, Q_R \), and the function, \( V(Q_1,Q_{-1}) + az \), represents the consumer’s general utility function for \( Q_1 \), \( Q_{-1} \), and \( z \), his part of the total numeraire output, \( Z \). The slope of the indifference curve is given by \( V_{Q_1}/a \), where \( a \) is the consumer’s constant.
marginal utility for \( z \). Summing the analogous indifference curves of the successively lower-demand-price customers, we obtain the generally much-steeper indifference curve of Figure 2. The slope of the curve at a given \( Q_1 \) is the slope of the original curve, \( V_{Q_1}/a \), times \( f_i(N_i) \).\(^2\) For example, if \( i \) had 2 customers, the better one having a MRS twice that of the other so that \( f(2) = 3/2 \), the slope of the Figure 2 iso-utilities function facing the retailer would be \( 3/2 V_{Q_1}/a \). Since this slope is also the sum of the \( N_i \) individual MRS's, it is also the same marginal social value that appears in equation (2); that is, \( U_1 f_1(N_1)/A = V_{Q_1} f_1(N_1)/a \). With \( Q_{-1} = 0 \), we can superimpose \( i \)'s cost function, \( C_1(Q_1) \), on Figure 2 by drawing a horizontal line over from \( \Sigma z_o = Z(0) \) and then noting that \( C_1(Q_1) \) is \( Z_o - Z(Q_1) \), the

\(^2\)Since the ratios of the individual MRS's implied by \( f_i(N_i) \) are independent of \( Q_i \), any individual \( V \) function is a simple linear function of any other \( V(\cdot) \) function.
amount of numeraire that must be sacrificed to produce $Q_1$ units of 1. The slope of this curve is obviously $-C'_1(Q_1)$. The producer's optimum for a given $N_1$ occurs at a $Q_1$ such that the slopes of the two curves in Figure 2 are equal, i.e., where $C'_1(Q_1) = V_{Q_1} f_1(N_1)/a = U_{Q_1} f_1(N_1)/a$.

While the same condition obviously holds for any positive --- but fixed --- $Q_{-1}$ purchased by 1's customers, we cannot infer that the social optimality condition given in equation (2) is satisfied under laissez faire calculation because retailers can affect the $Q_{-1}$ choices of their customers, and this in turn affects the retailers' returns to altering their $Q_1$ levels.

Allowing now the $Q_{-1}$-vector to be positive and noting that the $i^{th}$ retailer cannot affect the qualities produced or prices asked by the retailers under Bertrand interaction, we can immediately see that the $i^{th}$ retailer can affect other retailers only by influencing his customers' decisions on whether
or not to purchase the given outputs produced by the other retailers. We assume throughout that consumers observe all retailers' price-quality decisions prior to making any of their consumption purchases. Let $P_{-i}$ be the vector of unit prices charged by the other retailers to $i$'s highest-demand-price customer and $Q_{-i}(Q_i)$ the corresponding vector of quantities purchased of the other retailers' overhead services by this consumer. The point $Z_o - P_{-i}Q_{-i}(0)$ on the vertical axis of Figure 3 then represents the quantity of numeraire possessed by the consumers once they have made their privately optimal expenditures on other retailers' goods, given that retailer $i$ has produced a zero output. The highest-demand-price consumer's utility level at $Q_i = 0$, $V(0, Q_{-i}(0)) + a(z_o - P_{-i}Q_{-i}(0))$, is constant along all points on the scalloped iso-utility curve drawn from $Z_o - P_{-i}Q_{-i}(0)$ on the vertical axis. The scallop points represent the effect of the consumer's dropping-off substitutes or adding-on complements as $Q_i$ expands. Utility remains unchanged at these switch points because changes in $Q_{-i}$ are matched by changes in consumer expenditures of the numeraire at these points. As in Figure 2, the $i$th retailer's total revenue is the distance between the iso-utility curve and the horizontal line drawn out from a point on the vertical axis (in this case, the point $Z_o - P_{-i}Q_{-i}(0)$). The iso-utility curve thus represents a total-benefit-from-$Q_i$ curve rather than a more familiar, $Q_{-i}$-constant iso-utility curve. As $Q_i$ increases from zero, we follow down the consumer's ordinary, $Q_{-i}(0)$-constant, indifference curve until the consumer is just willing to change from $Q_{-i}(0)$, his initial set of purchases from other retailers. At that point, as we have said, consumer utility does not change in that any would-be utility jump is matched by a utility-maintaining expenditure jump, the continuity of $V(.)$ assuring us that utility does not jump merely because of the changes in $Q_i$. Since the prices and qualities of the other retailers
are given to the \( i \)th retailer, reaching the switch point induces the consumer
to discretely increase his purchases of \( Q_1 \)-complements or decrease his
purchases of \( Q_1 \)-substitutes. In either case, the marginal value of \( Q_1 \) jumps
to higher levels at the switch point, thereby shifting up the absolute value
of the slope of the new \( Q_1 \)-constant indifference curve at \( Q_1 \) levels beyond
the switch point, thus producing the scalloped iso-utility curve of Figure 3.
As in Figure 2, \( C_1(Q_1) \) is represented by a distance from the horizontal line
drawn out from \( Z_0 - P_{-1}Q_{-1} \) on Figure 3. The vertical distance between this
curve and our iso-utility curve at any given \( Q_1 \)-level thus represents the \( i \)th
retailer's profit at this \( Q_1 \)-level.

\[ f(N_1^0)[V(Q,Q_{-1})] + a(Z_0 - P_{-1}Q_{-1}(Q_1)) = f(N_1^0)[V(Q_1,Q_{-1}(0))] + a(Z_0 - P_{-1}Q_{-1}(0)) \]
The graphs can now be used to demonstrate the impossibility of achieving the social optimum and, for that matter, any pure strategy equilibrium, with more than one active retailer selling related services to a given set of customers under perfect price discrimination. First, a profit-maximizing \( Q_i^* \), labelled \( Q_i^* \), obviously occurs only at the interior of a scallop. At an end point of a scallop, the firm can obviously increase profits by either expanding or contracting its \( Q_i \)-level. But a contradiction immediately arises. If there are other retailers selling related services to \( i \)'s customers at positive prices that lead the \( i \)th retailer to choose an interior point on a scallop, some of these retailers have set too low a price for their given services! Active sellers of complements, which are necessarily switched-into at \( Q_i \) outputs less than \( Q_i^* \), could charge \( i \)'s customers higher prices without losing these customers because the hypothesized level of \( Q_i \), being greater than the switch-point level, implies a higher value of \( Q_i \)-complements than exists at the lower switch point. Similarly, active retailers of neighboring \( Q_i \)-substitutes, which are necessarily switched-away-from only at \( Q_i \)-levels discretely beyond \( Q_i^* \), are under-pricing their services because somewhat higher prices to \( i \)'s customers would not cause these buyers to quit buying these services. Since producers of complements will obviously not choose prices with switch points beyond \( Q_i^* \) while producers of substitutes would similarly not pick prices generating switch point quantities below \( Q_i^* \), rational pricing by any active supplier of a related retail service would generate a switch point at \( Q_i^* \), directly contradicting \( i \)'s maximization requirement that \( Q_i^* \) be an interior rather than a switch point.

Note that if we did not allow the \( Q_{-i} \) sellers to adjust their prices to the expected \( Q_i \)-choice of the \( i \)th retailer, simply assigning price a la Lindahl, the above non-existence argument would fail. We could assign lump-
sum price components to the retailers so that the switch-points do not even appear. Each firm's profit-maximizing $Q_i$-choice would then obviously satisfy the second optimality condition and a Lindahlian optimum would be achieved. However, as we have been arguing, this is mere calculation. Lindahl pricing is not privately rational and should not be the expected outcome of private, decentralized, pricing decisions.

Our impossibility results should not cause alarm. If the complete information required to achieve a Bertrand-type, perfectly discriminating solution were available to the producers, a market system would be unnecessary. A centrally planned solution would suffice. So all the results say is that a market system cannot achieve an optimum when the system is, at best, redundant. Analogous impossibility results hold for all-private-goods economies in which Bertrand suppliers have increasing marginal costs.\(^3\) The

\(^3\)Our proof of the inability of increasing-cost Bertrand rivals to achieve a social optimum, a straightforward generalization of the arguments of Kahn and Alger, is a one-liner. Consider a fixed group of increasing-cost, Bertrand-interacting suppliers of a homogeneous private good: If prices were all at the efficient, Walrasian level, any seller would gain by raising his price to the inefficient, monopoly price to the demanders who are left unsatisfied by the other sellers, thereby upsetting the Walrasian solution.

Completing the analogy to our above collective-good result, any Bertrand-type, pure strategy solution under increasing costs is strictly impossible when the sellers have complete information. Our proof here is by contradiction: suppose that a given set of prices by the sellers describes an equilibrium solution. First, some price differences must exist: If all prices were the same, the common price would have to be higher-than-Walrasian because we have just seen that it cannot be equal to the Walrasian price and because any seller would obviously raise his price if the common price were sub-Walrasian. But, at a higher-than-Walrasian common price, it pays a supplier to shade the price and produce at marginal cost instead of a rationed-back level of output. So price differences must characterize the hypothesized equilibrium set of prices. Now a lowest-price seller cannot be rationed-back; he sells an output at which price equals marginal cost because he could always sell to the customers of the high-priced sellers if he wished to do so. Therefore, a price increase by any given lowest-price seller would not force the seller to lose customers to other lowest-price sellers. Consequently, this seller could compensate himself for any loss in sales to his existing, positive-surplus customers by increasing his sales to the customers
general idea, then, is that laissez faire competition cannot be the socially optimal institution represented by economists when it is a simultaneous, i.e., Bertrand-type, pricing process. Even the mere existence of an equilibrium in a simultaneous-pricing environment requires us to introduce special, hopefully realistic, informational imperfections into the discussion. Since the assumption that consumer demands are somehow known a priori by Bertrand-type sellers is both unrealistic and inappropriate to the evaluation of a market system, it is the most natural assumption for us to drop. Fortunately, dropping this assumption will substantially remove the above existence problem, at least for our collective goods suppliers. The point is developed in the following section.

III. The Tendency of Retailers To Overproduce Overhead Services Under Realistically Imperfect Information

A. Pricing under realistically imperfect information. When Bertrand retailers do not have sufficient information to perfectly price discriminate, they must gamble on their potential customers' true demand prices, which are, to them, random variables. Let \( \Pi_i^\alpha(P_i;Q_i,\text{EQ}_i^\alpha(Q_i)) \) represent the \( i \)th retailer's subjective probability that the \( \alpha \)th consumer will buy his given \( Q_i \) at a price of \( P_i^\alpha \), where \( \text{EQ}_i^\alpha(Q_i) \) is the \( i \)th retailer's expectations of the vector of \( \alpha \)'s purchases of other, available overhead services. We assume that \( \Pi_i^\alpha(.) \) is differentiable with \( \exists \Pi_i^\alpha/\exists P_i^\alpha < 0 \) and that the retailer's expected revenue from customer \( \alpha \), \( Q_i P_i^\alpha \Pi_i^\alpha(P_i;Q_i,\text{EQ}_i^\alpha(Q_i)) \), reaches a maximum at a unique value of \( P_i^{\alpha^*} \) and thus \( \Pi_i^\alpha^* \) for each customer \( i \), given \( Q_i \). (This occurs where the price elasticity of \( \Pi_i^\alpha \) is -1.) Any such \( P_i^{\alpha^*} \) solution must obviously exceed zero.

of higher-priced sellers. Since it always pays sellers to change their prices from the hypothesized equilibrium set of prices, the hypothesis is contradicted.
Note that a fixed subsidy to the purchase of the retailer's sales -- which we may empirically identify with an ad valorem subsidy to the sales of his private-good complement -- would, by adding \( s_1 \Pi_1^\alpha \) to the retailer's expected revenue, always increase \( \Pi_1^{\alpha*} \) and lower \( P_1^* \) for a given \( Q_1 \) in the same way that a fixed per-unit subsidy to a classical monopolist increases the monopolist's output and lowers his price for a given quality of his output. And, similarly, sufficiently high subsidy rates would induce the seller to charge negative prices as he would be induced to pay negative use-value customers to use his services in order to increase his subsidy revenue. Of particular interest will be the subsidy rate, \( s_1^\infty \), that makes the retailer's optimal price to the consumer zero. (This rate is easily seen to be equal to \( \Pi_1^\alpha/\Pi_1^{\alpha'} - 1 \).)

B. Socially optimal \( N_1 \)-subsidies. In the absence of such subsidies, the expected number of customers under laissez faire, \( N_1^* = \sum \Pi_1^{\alpha*} \), is obviously less than our socially optimal number, \( N_1^o \), because some potential customers with positive marginal values are excluded while no customer with a negative marginal value is included in \( i \)'s consuming group. A socially optimal policy response to this inefficiency assures that any buyer with a positive use-value purchases the retailer's overhead service and a potential buyer with a negative use-value does not. The commonly mentioned government policy achieving this result is the forcing of all retailers to charge zero prices to all buyers, heavily subsidizing the retailers-turned-government-suppliers according to the qualities of their overhead services. However, this policy creates administrative difficulties in monitoring prices and qualities, making this "governmental provision" system expensive, more socially expensive, we assume, than a market-oriented alternative in which the government cheaply achieves an efficient \( N_1 \)-subsidy by exploiting the discriminatory, demand-
revealing process utilized by private retailers in marketing their overhead services. As discussed above, a simple form of this free-market exclusion process has retailers charging uniform, positive markups on certain, complementary, private goods. An optimal \( \eta_1 \)-subsidy would, correspondingly, subsidize the quantities sold of these private-good complements, acting as the simple subsidy discussed above. Relatively high use-value consumers of the overhead services of a given retailer — who are much more likely to buy the retailer's private-good-complement than are low-use-value consumers — are thereby subsidized at much higher rates than low use-value consumers. And a sufficiently high subsidy level would induce the retailer to offer an effectively zero price to every class of consumer.

We shall argue, in Section IV below, that U.S. retailers typically are subsidized in just this way.

The fixed, socially optimal, subsidy rates, denoted \( s_1^{\infty} \), in order to do their job in inducing firms to offer their overhead services to consumers at effectively zero prices, must anticipate the retailers' quality decisions.

C. Equilibrium qualities of the overhead services. To determine the firm's privately optimal quality choice, \( Q_1 \), which we also want to compare with the socially optimal \( Q_1 \) described in equation (2), we simply have the firm vary its \( Q_1 \) so as to maximize its expected profit,

\[
\sum_{\alpha} (P_{11}^\alpha Q_1 + s_1^{\infty}) \pi_{11}^{\alpha^*}(P_{11}^\alpha; \text{EQ}_1^\alpha(Q_1)) - C_i(Q_1),
\]

holding each \( \pi_{11}^{\alpha} \) constant at its privately optimal level by suitably varying \( P_{11}^\alpha \) with \( Q_1 \). This generates the following first order condition for \( Q_1 \):

\[
\sum_{\alpha} (P_{11}^\alpha + Q_1 \frac{dP_{11}^\alpha}{dQ_1}) \pi_{11}^{\alpha^*} = C_i'(Q_1),
\]
or, since $s_1^\alpha$ and thus $p_1^\alpha$ and $\Pi_1^\alpha$, are at socially optimal levels, simply

\begin{equation}
Q_1 \sum_{\alpha} \frac{d\Pi_1^\alpha}{dQ_1} \bigg|_{\Pi_1^\alpha \text{ con}} = C'(Q_1).
\end{equation}

Each derivative in the sum on the left side of (4) is easily determined by solving the equation defining it; viz.:

\[0 = \frac{d\Pi_1^\alpha}{dQ_1} = \frac{\delta \Pi_1^\alpha}{\delta p_1^\alpha} \frac{dp_1^\alpha}{dQ_1} + \frac{\delta \Pi_1^\alpha}{\delta Q_1} + \sum_{j \neq 1} \frac{\delta \Pi_1^\alpha}{\delta EQ_j} \frac{dEQ_j}{dQ_1}.\]

I.e.,

\[\frac{dp_1^\alpha}{dQ_1} \bigg|_{\Pi_1^\alpha \text{ con}} = -\frac{\delta \Pi_1^\alpha}{\delta p_1^\alpha} \frac{dp_1^\alpha}{dQ_1} - \frac{\delta \Pi_1^\alpha}{\delta Q_1} + \sum_{j \neq 1} \frac{\delta \Pi_1^\alpha}{\delta EQ_j} \frac{dEQ_j}{dQ_1}.\]

(5)

By holding $\Pi_1^\alpha$ constant, we are holding constant the utility that the $i$th retailer expects he is providing the $\alpha$th customer. Thus, using the previous section's notation to express (5) in terms of individual utility functions, the retailer's expected marginal revenue from $Q_1$ expressed in (4) is:

\[\sum_{\alpha} \Pi_1^\alpha \frac{dp_1^\alpha}{dQ_1} \bigg|_{\Pi_1^\alpha \text{ con}} = \sum_{\alpha} \Pi_1^\alpha \left[ \frac{\hat{\nu}_{Q_1}^\alpha}{a} + \frac{\hat{\nu}_{Q_1}^\alpha}{a} a \frac{dEQ_1^-}{dQ_1^-} \right],\]

where $\frac{\hat{\nu}_{Q_1}^\alpha}{a}$ represents $i$'s estimate of $\alpha$'s MRS for his service quality.
It is plausible to assume that the retailer knows the above solution marginal revenue with certainty through past experiments with small changes in $Q_1$ because such changes are of negligible cost to the retailer. Assuming further that each consumer's actual MRS is an unbiased estimate of the producer's expectation of this MRS, the expected marginal revenue in (6) can be written:

\[
\sum_{\alpha} \left( \Pi_{1}^{\alpha} \frac{dP_{1}^{\alpha}}{dQ_{1}} \right) = \sum_{\alpha} Q_{1} \frac{dP_{1}}{dQ_{1}} = \sum_{\alpha} \frac{V_{Q_{1}}^{\alpha}}{\alpha} + \frac{V_{Q_{1}Q_{1-1}}^{\alpha}}{\alpha} dQ_{1},
\]

where \(\{N_{1}^{0}\}\) denotes the equilibrium set of actual customers of the \(i^{th}\) retailer.

To further compare this problem to the perfect information case, Figure 4 displays the retailer's $Q_{1}$-optimization problem on a graph analogous to Figure 3, where the retailer's total revenue curve again has each $P_{1}^{\alpha}$ vary with $Q_{1}$ so as to maintain the producer's expectation of the solution levels of utility generated by his service. Beginning the fixed subsidy, $s_{1}^{o}$, at $Q_{1} = 1$, expected revenues now begin at $Q_{1} = 1$, being $(P_{1}(1) + s_{1}^{o})\Pi_{1}^{*}$ at that point, where $P_{1}$ is the set of prices for all consumers, $\alpha = 1, \ldots, N$, and $s_{1}^{o}$ and $\Pi_{1}^{o}$ are, respectively, the set of optimal $N_{1}$-subsidies and correspondingly optimal retailer use-probabilities for all consumers. (Because $P_{1}(Q_{1})$ increases with $Q_{1}$ and is zero in the equilibrium, $P_{1}(1)$ must be strictly negative.) Since this total revenue curve, and in particular the $P_{1}(Q_{1})$-function, is constructed so as to leave $\Pi_{1}$ --- and thus the producer's expectation of the consumers' utilities --- constant, it is also a kind of iso-utilities curve. The equilibrium, $P_{1}^{\alpha} = 0$, slope of this iso-utilities curve is given in equation (7).
Figure 4

Two important differences appear between Figure 4 and the iso-utilities curve of Figure 3. First, the switch points are gone. When $Q_1$ increases, the $i$th retailer, who now never knows for sure whether a given consumer is going to buy any seller's output, alters only his prior probabilities, $\Pi_{-1}$, that consumers are going to buy the related services offered by other retailers. The source of the existence problem — viz., the desire of other retailers to raise prices until other retailers border on switch points — is removed by smoothing out the effects of such anticipated price increases on
the expected sales of the other retailers. At the same time, the effect of changes in \( Q_i \) on the expected levels of \( Q_{-i}^\alpha \) and thus on the demand price for \( Q_i \) becomes a continuous effect relevant to the retailer's computation of his private marginal value of \( Q_i \). Thus, as described in equations (5) through (7), the retailer's expected revenue from a quality improvement contains an extra term beyond the consumer's ordinary MRS, a term expressing the expected increase in the consumer's demand-price for \( Q_i \) caused by alterations in the probabilities of his purchases from other retailers induced by his unit increase in \( Q_i \).

The second important difference between the two iso-utilities curves is that Figure 4 is simply the producer's expectation of the combination of \( Q_i - Z \) points that will, under his pricing scheme, leave his customers indifferent. It has taken rational expectations assumptions to connect these expectations to the underlying reality in order to evaluate the social efficiency of the retailer's equilibrium \( Q_i \)-choices. In particular, regarding the level of the expected revenue function, recall that, whatever the firm's subjective \( H_i \)-levels, the optimal subsidy induces the seller to charge an efficient, zero price to each consumer. (Empirically, as long as observed subsidy rates paid directly to the consumers approximate the observed, retailer/discounter price differential, this is a realistic assumption.) Similarly, regarding the slope of the function, we have assumed -- fairly realistically we have argued -- that the retailer: (a) knows the actual near-equilibrium revenue from a small quality change when his prices increase so as to leave his potential customers' expected purchases from him unaltered, and (b) has an unbiased estimate of each customer's near-equilibrium marginal real use-value for his service.
The marginal overvaluation and resulting overproduction of $Q_i$ follows immediately upon substituting equation (7) into equation (4) and recalling that each term in the vector product, $\frac{\nu_{Q_i Q_{-1}}}{a} \cdot \frac{dE_i}{dQ_i}$, is nonnegative, being positive for related services and zero for unrelated services. Thus, the quality-expanding retailer, whether "rent-seeking" by reducing the expected sales of substitute-producing retailers or "free-riding" by gaining from the expanded sales of complementary retailers, sees too large a private return from his expansion.

D. An Illustration. The following special case may aid our understanding of the above, rather abstract, uncertainty model. Assume that the various total real-use values for the $i$th retailer's output, i.e., the various integrals of the intramarginal MRS's between $Q_i$ and $z$ for a given set of individual $Q_{-1}$ purchases, vary from a high of $\bar{V}_i(Q_i; Q_{-1})$ to a low of $\underline{V}_i(Q_i; Q_{-1})$, the latter being negative for all feasible values of $Q_i$ and $Q_{-1}$. The percentage of consumers with total real use values above any given value, $G_i$, where $0 < G_i < \bar{V}_i$, is given by

$$\frac{\bar{V}_i - G_i}{\bar{V}_i - \underline{V}_i}.$$

Retailers know this cumulative frequency distribution but have no information about the individual values of the various consumers. The best pricing policy a retailer can adopt is therefore to set a single, nondiscriminatory, total price, $G_i = P_i Q_i$ for his overhead service, given $Q_i$, the quality of his overhead service, and $Q_{-1}$, his correct $(R-1) \times N$ expectation of the set of total services that each of his potential customers will be buying from other retailers. With the proportion of consumers having use values above this price given by the above distribution function, we can express the retailer's profit in (3) as
(3') \[ (G_1 + s_1) N \cdot \left( \frac{\bar{V}_1 - G_1}{\bar{V}_1} \right)^2 - C_1(Q_1). \]

Setting the price derivative of (3') equal to zero, we find its profit maximizing level to be:

\[ G_1^* = \frac{\bar{V}_1 - 2s_1}{3}. \]

With \( s_1 = 0 \), \( G_1^* = \frac{\bar{V}_1}{3} \). Since this value is positive, inefficient over-exclusion exists with \( s_1 = 0 \). Setting \( s_1 \) so that \( G_1^* = 0 \) and assuming \( Q_1 \) has been set at its optimal level, \( Q_1^* \), we see that \( s_1^* = s_1^0 = \frac{\bar{V}_1}{2} \).

While this subsidy generates socially optimal utilization of \( i \)'s overhead services, it rewards him too much for being in business. For the social value of his product, the average of the positive values for his service, which is easily calculated to be \( \frac{\bar{V}_1}{3} \) under the assumed distribution, than his unit subsidy of \( \frac{\bar{V}_1}{2} \). License or entry fees could be introduced to solve this problem; we are formally avoiding the problem here by simply assuming a fixed number of retailers, \( R \). Nevertheless, our eventual tax on the firm's quality will work to eliminate this too-many-firms characteristic, which persists in related examples, and may thus completely eliminate the above need for license fees.

Note that our solutions for the conventionally discussed laissez-faire case in which \( s_1 = 0 \) can never possess this too-many-firms property, even when we take as given the suboptimal number of customers present when \( s_1 = 0 \). Thus, in our example, the \( i \)th retailer's per-customer revenue \( \frac{\bar{V}_1}{3} \), is far less than the aggregate or his customers' values for his unique product, the lowest of which is \( \frac{\bar{V}_1}{3} \).

The above, subsidy-influenced choice of \( G_1 \) determines a socially optimal utilization rate, \( N_i^0/N = (\bar{V}_i/\bar{V}_1 - \bar{V}_1)^2 \), for the \( i \)th overhead
service. Keeping utilization constant at $N_{1}^{0}$ by suitably varying the total
price with $Q_{1}$ around its optimal level, the $Q_{1}$-derivative of the $i^{th}$
retailer's profit function in (3') is zero when

\[ (5') \quad N_{1}^{0} \left. \frac{dG_{i}}{dQ_{1}} \right|_{N_{1}^{0}, Q_{i-1} \text{con}} + N_{1}^{0} \left. \frac{dG_{i}}{dQ_{i-1}} \right|_{N_{1}^{0}, Q_{i} \text{con}} \frac{dQ_{i-1}}{dQ_{1}} = C_{i}'(Q_{1}). \]

Since the first term on the left above is simply the marginal social value of
$Q_{1}$ described in (2), the second term on the left, which is again always
positive as long as some of the other retailers' overhead service are related
in consumption to $i$'s, again describes an unambiguous private overvaluation
of $Q_{1}$.

IV. $N_{1}-$subsidies in the Real World

Since the private overvaluation of the quality of the overhead services
provided by retailers is unambiguous only when socially optimal $N_{1}-$subsidies
are in place, we will be unable to evaluate observed economic policies system-
atically attenuating the property rights of relatively high quality retailers
unless we can find a set of such $N_{1}-$subsidies in the real world, at least for
certain types of retailers.

It will be helpful to separate all private-good complements sold by
retailers into three mutually exclusive and exhaustive categories: (1) Goods
consumed while outside the consumers' body; (2) Goods consumed while on the
surface of the consumer's body, and (3) Goods consumed entirely inside his
body. This seemingly irrelevant taxonomy will help us isolate various kinds
of real-world externalities.

Goods consumed outside a person's body (e.g., appliances, autos, houses,
e tc.) are "coveted"; then can be taken by foreign aggressors and hence gener-
ate a negative defense externality. The greater the stock of such goods, the
greater the national defense effort required to provide a given level of national security. But these goods are not taxed to reflect this externality! Consumer durables offer a unique exception in the U.S. tax system, which efficiently taxes all other forms of coveted capital at approximately the same rate (Thompson, 1974). With consumer durables uniquely relieved of their national defense tax liabilities, we have the implicit subsidy we are seeking. Moreover, the average, point-of-sale, U.S. capital tax, about 15%, roughly approximates our casual estimate of the typical retail mark-up over and above the cost of ancillary private-good services (such as inventory services, flattery and related psychological services, and "free-trial" services). Expensive items, in particular houses and cars, have directly observable dealer-mark-ups and brokerage commissions. These mark-ups, being only about 5% to 10%, appear to make the 15% implicit subsidy too large. However, since these expensive items are typically resold through dealers and brokers, 15% may be a reasonable estimate of the present value of the sequence of apparent over-markups on a typical, expensive, consumer durable good by information sellers using this private-good complement to collect for their information.4

4One might regard positive mark-ups on used goods as providing original purchasers with too little resale incentive. However, the ubiquitous over-incentive to transact to current sellers who believe that future market prices will be less than the prices expected by current buyers (Thompson, 1966, Hirshleifer, 1971) rationalizes the traditional on governmental support of resale contracts allowing brokers positive mark-ups on complementary private goods rather than fixed service fees.

The same holds, of course, for resales of rights to durable producer assets (e.g., common stocks), where observed governmental policies induce security as well as real estate brokers to charge ad valorem commissions rather than flat service fees. (How common-stock transaction taxes fit into an optimal capital tax system is discussed in Thompson, 1974.) At the same time, governmental policies allow those brokers to collectively fix their nominal commission rates at common levels, thereby facilitating exclusion by these information-providers in the same way that private-resale-price-maintenance contracts allow retailers selling newly manufactured goods to protect themselves from free-riders. Moreover, the policy-analogy is complete
Our second kind of private-good complement, being consumed on the physical surfaces of individuals, are highly tailored to the individuals and not significantly coveted by foreign aggressors. These goods — like clothes, cosmetics, haircuts, etc. — serve, however, as "adornments" in our society in that they are consumed largely to make the consumer more "attractive" to others. While the benefits created for his friends are largely internalized by the consumers of these goods, the costs imposed on others — though the numerous decreases in the utilities of the friends of the now-relatively-less-attractive individuals — are not internalized.

I call such decreases in utility, resulting as they do from simple increases in the quality of the services consumed by other consumers, "relative quality externalities" (see Thompson-Canes). They are not Veblen effects. "Veblen, and later Galbraith, argued that the elaborate consumer durables found in many houses and vehicles were too elaborate because of the effect of quality improvements of such goods on the utilities of other individuals. These authors, however, failed to distinguish "pecuniary" externalities from direct, "technological" externalities. As houses, autos, and yachts are typically shared with one's friends, an increase in the quality of such possessions serves to increase the owner's ability to attract quality friends and thereby injures neighboring suppliers by making it more expensive for them to maintain their previous supplies of friendly associations. Because such a loss is only "pecuniary" in that it implies a corresponding gain to quality friends, there is no net external effect of these quality improvements on
economic welfare. Thus, economists have been wrong in automatically accepting the "keeping-up-with-the-Jones'" effect as a real externality. The long-term trend toward increasingly elaborate consumer durables relative to per capita income is simply a reflection of our steadily increasing scarcity of friends relative to manufactured products and therefore a steadily increasing real price of friends.

While we have yet to devise a general method of quantifying the relative quality externality in order to see if actual retail mark-ups on "adornments" match the corresponding external diseconomies, the one empirical study we have done so far (Thompson-Canes) does suggest a remarkable ability of governments to devise an optimal response to this problem. In view of this, and in the absence of any evidence to the contrary, we merely assume that the relative quality externality for typical adornments approximates the typical retail mark-up on the good.

This rough approximation allows us to empirically apply our unambiguous overvaluation result, which implies the optimality of economic policies limiting the appropriability of high-quality retailers of private-good complements falling into one of our first two categories. For either kind of good, there should be policies restricting the appropriability of high-quality retailers.

Remarkably, such restrictions are quite common throughout the developed world. In the U.S., we have laws preventing resale price maintenance contracts, whose main purpose is obviously to insure retailers against the free-riding discounts of retailers. Similarly we have a whole complex of anomolous laws [e.g., laws restricting exclusive dealerships, exclusive territories, price discrimination across retailers, etc. (see Schwartz-Eisenstadt)], serving society only by limiting the appropriability of high-quality
retailers. Also, with certain important exceptions described below.\textsuperscript{5} England shares the same kind of policy.

The only era of widespread, legalized, resale price maintenance in the U.S. began in the late 1930's, after "cut-rate" depression discounters began to allow the increasingly mobile consumers to substantially "free-ride" on the services of relatively high-quality retailers (Overstreet). The absence of implicit national-defense and subsidies the lesser significance of adornment externalities during this early period meant that policies encouraging the complete appropriability of retailer-provided benefits could be desirable (in a second-best sense) through their positive effects on the numbers of retailers. An argument partially rationalizing the continuation of our resale-price-maintaining, "Fair Trade" laws throughout the 50's and 60's — after our national-defense subsidies and adornment externalities grew to significance — will be supplied only after we have considered our third, and final, type of private-good complement.

Private-goods safely consumed entirely within the body — like food, pharmaceutical drugs, and popular books and periodicals — being neither coveted by foreign aggressors nor capable of making other individuals look less attractive, generate no obvious external diseconomies and therefore receive no indirect subsidy from the U.S.'s laissez-faire policy toward safe

\textsuperscript{5}An important caveat is that current laws may also work to prevent resale price maintenance and exclusive distribution agreements when they would be employed to achieve other socially desirable, private goals. In particular, resale price maintenance may be used to provide small retailers with a means of making price commitments to protect their margins against overspending, specific-service-receiving customers and exclusive distribution may be used as part of a labeling service provided by high-quality, high-rent-district retailers.
consumer goods. However, goods in this third category that are also "perishable", or short-lived in their influence on the body, call for very little in the way of retailer provision of collective-good information. Such goods are simple, nondurable, "experience goods" in the terms of Nelson and can be individually evaluated by each consumer through a simple, inexpensive sampling of the goods. Hence, food "retailers" provide essentially no information regarding the qualities of their various goods and corresponding charge no noticeable mark-ups over their private-good costs, the cost of inventory-holding and contracting. In contrast, goods in this category that are durable in that they may easily have a relatively long-term influence on the body create a special problem. The only goods we could find in this category (see Nelson's list of "experience-goods") are pharmaceutical drugs and books and periodicals. Here, we find a singular exception to the above pattern in that while retailers should generally supply consumers with significant amounts of information concerning the durable-effects of such private goods, consumers receive no apparent subsidy for purchasing the goods.

Summarizing to now, we have, with only one exception, found significant implicit subsidization of private-good-complements where such subsidization is efficient. The exception is an absence of subsidization of effectively durable goods consumed entirely within the body, the only apparent examples in this category being pharmaceutical drugs and popular books and periodicals. Correspondingly, our theoretical overvaluation results tell us that we should attenuate the property rights of all high-quality retailers except

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6Goods that present the consumer with a risk of loss of bodily function or death are overpurchased in that the consumer does not have to, and typically does not, compensate friends and relatives when he incurs such a loss. This "death externality" rationalizes the myriad of safety regulations we see throughout the developed world (Thompson, 1979.)
information—providing druggists and sellers of books and periodicals.

In fact, U.S. druggists and book sellers were the chief political forces behind the establishment and maintenance of the U.S. Fair Trade laws from the 1930s through the 1960s (Overstreet). These laws, which gave quality retailers the ability to prevent free riding by discounters were only recently eliminated only after druggists and booksellers lost much of their usefulness as relatively informed suppliers of information to their customers. Still more remarkably, in England, where booksellers and druggists have succeeded in maintaining their traditional role as informed advisors, Fair Trade protection is afforded only to booksellers and druggists!
References


