EFFICIENT LABOR CONTRACTS WITH EMPLOYMENT RISK

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Introduction

The employment contract literature is founded on the observations that worker-firm ties are often long lived, yet subject to transitory variations in hours worked and compensation. The early literature focused on specificity of human capital (Oi, 1962; Becker, 1974) and transactions costs (Rosen, 1968) as stabilizing employment relationships. Later, emphasis shifted toward opportunities for risk sharing between workers and their employers, and work on the terms of "implicit" employment agreements began. At this date, the analysis of income risk pooling in employment contracts so predominates the literature that recent contributions and surveys (e.g., Rosen, 1983) are devoted almost exclusively to this problem. In contrast, in this paper we analyze the form of efficient employment agreements when the demand for consumption insurance can be satisfied by non-contractual means. We assume workers can save and this assumption breaks the sometimes unstated identity between current earnings and consumption that motivates most recent contributions. Here, the relevant constraint on consumption choices is the worker's wealth and consumption is determined by a strong form of the permanent income hypothesis.

To the observations of longevity of employment and transitory variation in terms of trade we add that, subject to preannounced rules of compensation, changes in hours or days worked are normally determined unilaterally by the firm. Rather than the stylized classroom model of a market determined wage or wage schedule, against which the worker is free to choose hours worked, the institutional fact is more likely to be a tied wage-hours offer that includes provisions for variance. The model we develop rationalizes this and other common features of employment agreements.

In our analysis, firms experience labor demand shocks and optimize employment against labor costs. Simultaneously, workers choose a consumption
policy that is constrained by permanent income. We show the existence of a unique compensation schedule that guides decisions of both parties. The employer cost/worker compensation schedule has the properties that (i) when employers maximize profits against this schedule, efficient employment outcomes are generated without need for third-party enforcement or monitoring of demand states; and (ii) when workers are compensated along this schedule they choose a consumption policy that leaves them indifferent among realizations of labor demand, given their knowledge of its distribution. State indifference is a consequence of the worker's consumption-saving policy: the utility that is equalized is the myopic utility of current consumption and leisure plus the utility-equivalent value of net savings.

An implication of worker demand state indifference is that the employment levels chosen by the firm satisfy the necessary conditions of a dual problem. In this problem the firm presents the worker with the previously derived labor cost/compensation schedule, and the worker is allowed to choose either a unique level or a distribution of employment. This choice results in a partition of all possible employment distributions into two sets: a preferred class of distributions among which the worker is indifferent, and an inferior class. The firm's optimal employment distribution is shown to be one member of the worker's preferred class. Since the firm's optimal distribution is unique and within the workers preferred set, it is both costless and efficient for workers to delegate the hours decision to the employer.

The next section sets out the basic framework for the problem and solves for optimal compensation, consumption, and labor supply in response to varying demand for labor's services. The proposed contract is shown to be consistent with widely observed characteristics of employment agreements. Section 2 explores further properties of efficient employment contracts, focusing on
workers' attitudes toward employment risk and the feasibility of an efficient market equilibrium without contracts.

1. **Contracts With Uncertain Labor Demand**

   It is our view that a theory of employment agreements should be consistent with three stylized "facts" that are widely thought to characterize implicit of explicit employment relationships:

   **P1.** Employment and earnings vary over time within a job. When employment variation occurs, contractual earnings rise with time worked during a period. Earnings commonly increase at an increasing marginal rate, as with overtime premia or bonuses. Compensation may be positive even when no work takes place, as with supplemental unemployment insurance or severance pay.

   **P2.** Decisions to vary employment -- including overtime, reductions in hours, and layoffs -- are made unilaterally by the employer subject to mutually accepted rules for compensation. Workers have summary information _ex ante_ about employment and earnings prospects in a given job. This _ex ante_ information determines the equilibrium contract, but workers are passive participants in the agreement _ex post_.

   **P3.** Contractual reductions in employment and earnings, caused by declines in labor demand, are not preferred by workers.

   A fourth postulate -- that workers are risk averse -- might be added to this list. In fact, its implications have occupied such a prominent place in the thinking of economic theorists that almost the whole of the contract literature is devoted to it.⁶ We have no quarrel with the idea that workers wish to mitigate the consumption risks associated with uncertain income. Nevertheless, we know of no important empirical phenomena that require worker-firm risk shifting as an explanation, while formal empirical evidence on risk shifting as an important motive in labor contracts is both scant and largely negative (Brown, 1980).

   In contrast, a number of recent empirical studies of consumption and labor supply provide varying degrees of support for the implications of the life-cycle permanent income theory of consumption.⁷ There is strong evidence
that individuals manage their assets to smooth consumption relative to
transitory variations in earnings, even through periods of unemployment. As
an example, Hamermesh observed that half of all unemployment insurance
benefits are "...spent as if the individuals [receiving them] were fully able
to borrow or had sufficient savings to meet transitory losses of income
without any disruption in their consumption spending" (1982, p. 110). Access
to assets and short-term credit is likely to be particularly relevant for
individuals who are involved in stable, long-term employment relationships or
who experience only short periods of unemployment between jobs.

We therefore relax the assumption that only employers may insure workers
against income risks and analyze contracts that account for workers' private
consumption-saving strategies. For simplicity, we study the interaction
between a single worker and a risk-neutral employer who wish to maximize their
joint return. Productivity of the worker follows a random variable $\theta$, drawn
from $F(\theta)$ with density $f(\theta)$. The demand shocks represented by $F(\theta)$ are
viewed here as transitory, so labor mobility in response to permanent changes
is not a central issue. The value of production is $g(h; \theta)$ where $h$ is time
worked during a period. The function $g(\cdot)$ is concave in $h$ for fixed $\theta$ and
increasing in $\theta$. The worker's preferences are completely characterized
by a utility function, $u(C, T-h)$, defined over consumption, $C$, and leisure
or home production, $T-h$. At this point the only restrictions that we place
on $u(\cdot)$ are that it possess the usual curvature properties, i.e., convex
indifference curves, and that workers are risk averse in consumption;
$u_{11} < 0$. We also adopt the expected utility hypothesis which in this case is
an assumption that the worker's utility is linear in proportions of time spent
at various levels of employment and that the proportions may be either
probabilities or deterministic fractions.
To fix ideas we begin with a simple illustration, in Figure 1, of the value of the option to smooth consumption. Assume that the line, $U^*$, represents an indifference curve with a level of utility equivalent to the worker's highest valued alternative. The quantities $l_1$ and $l_2$ are chosen to be equi-distant from $l_0$. Now suppose the worker is permitted to choose between two jobs. One requires $T - l_0$ hours of work and pays $C_0$ every day. The other fixes hours at $h_1 = T - l_1$ and pays $C_1$ half the days and fixes hours at $h_2 = T - l_2$ and pays $C_2$ the remainder of the days.

Under the expected utility hypothesis it is tempting to conclude that the certain and uncertain prospects offer equal expected utility to the worker. They would if consumption were hand-to-mouth, that is, if there were no opportunities for saving. But if saving and dissaving is permitted it is clear that the worker cannot prefer the fixed hours-income relationship. So long as the marginal utility of consumption is not the same at $(C_1, l_1)$ as it is as $(C_2, l_2)$ the worker's utility is enhanced by transferring income from the low to the high marginal utility state. Thus for an arbitrary (non-degenerate) distribution of hours worked in a contract, expected utility will exceed $U^*$ if the payment schedule is made coincident with the indifference curve indexed by $U^*$.

Put somewhat differently, the illustrated indifference curve is one cost function for providing utility $U^*$ when consumption must equal income. Since costs cannot be increased when the consumption equals income constraint is relaxed, cost-minimizing compensation schedules will, in general, lie below $U^*$. Our analysis establishes that the worker is indifferent among all realizations of employment and income along the cost minimizing compensation schedule.
Fig. 1 — The worker's reservation utility level
1.1 Efficiency Conditions

Returning to the general issue, we can characterize efficient employment and consumption choices under the more common assumption that workers consumer only out of current earnings. This "hand-to-mouth" restriction is that \( c(\theta) = y(\theta) \) where \( y(\theta) \) is earnings in state \( \theta \). In this case employment, \( h(\theta) \), and earnings are chosen to solve

\[
\begin{align*}
\text{Max} & \quad \int \left\{ g(h(\theta); \theta) - y(\theta) \right\} f(\theta) d\theta \\
\text{subject to the expected utility constraint} & \quad \int u(y(\theta), T-h(\theta)) f(\theta) d\theta = U^* 
\end{align*}
\]

where \( U^* \) is a reservation level of worker utility determined by forces outside of the model, for example by labor market competition. Equations (1) and (2) are the common structure of contract models. The necessary conditions (pointwise for each \( \theta \)) are

\[
\begin{align*}
(3) & \quad g_1(h(\theta); \theta) = u_2/u_1 \\
(4) & \quad u_1(y(\theta), T-h(\theta)) = \lambda,
\end{align*}
\]

where \( \lambda^{-1} \) is a constant multiplier associated with constraint (2). Condition (3) equates the marginal product of worker's time to its shadow cost in terms of leisure or non-market production, which is the efficient allocation of time between activities. Condition (4) requires the marginal utility of consumption to be equalized across states of nature. These are standard to efficient transactions. Condition (4) is the modern definition of the permanent income hypothesis for consumption (e.g., Hall, 1978; Bewley, 1977).10 Notice that the efficiency conditions are not constrained by the state-specific link between consumption and income. All that matters is the permanent (expected) income, or wealth of the firm-worker pair.
A risk-pooling contract that satisfies (3) and (4) and that requires \( C(\theta) = g(\theta) \) must be viewed as a dubious description of observed employment relations for several reasons. First, note the implications for compensation in condition (4). Unlike most statements of the permanent income hypothesis, workers do not consume a constant proportion of wealth because of the non-separability of consumption and labor supply decisions at a point in time.\(^{11}\) Since income and consumption are the same, the condition implies that compensation increases in hours worked if, and only if, effort increases the marginal utility of consumption, i.e., \( u_{12} < 0 \). Otherwise, if leisure and consumption are marginal utility complements, income and labor supply are negatively related. We frankly know of no compensation schemes that offer higher earnings for less work, and we doubt that \( u_{12} < 0 \) is the reason.

Second, the contract stabilizes the marginal utility of consumption, not realized utility, so sometimes the worker's utility is greater than \( U^* \) and sometimes less. Related to our previous point, if \( u_{12} > 0 \) earnings and leisure are positively related and reductions in employment are preferred by workers.\(^{12}\) More generally, viewing consumption as a function of hours worked, the optimal contract implies \( c'(h) = y'(h) = u_{12}/u_{11} \). Therefore the change in realized utility as hours vary is

\[
\frac{du}{dh} = u_1 \left\{ y'(h(\theta)) - \frac{u_2}{u_1} \right\} = \frac{u_1^2}{u_{11}} \frac{d}{dy} \left( \frac{u_2}{u_1} \right).
\]

Equation (5) is negative if the marginal rate of substitution \( u_2/u_1 \) exceeds the marginal wage along \( y(h) \). Equivalently, a necessary and sufficient condition for (5) to be negative is that leisure be a normal good. Therefore, with normality of leisure and firm-provided insurance, efficient contracts imply that workers always prefer reductions in employment, even if earnings rise with hours worked.
In contrast to the worker's preference for employment reductions, the firm would always wish to increase employment beyond the efficient level. When leisure is a normal good (assumed in what follows), the ex post marginal cost of employment to the firm is less than the value of the worker's time \( y'(h) < u_2/u_1 \). Consequently, the efficient hours density that maximizes (1) requires monitoring to verify that (3) is satisfied, and so long as monitoring costs are positive, the firm may behave opportunistically by misrepresenting \( \theta \). Thus, the firm's unilateral discretion in setting employment will not support the solution given by (3) and (4). More realistically, we view \( \theta \) as summarizing an array of many factors that are difficult to verify in actual market situations, so contracts based directly on \( \theta \) are infeasible.

For these reasons we believe that the risk-shifting contract derived from (1) and (2) is a poor description of observed employment relations. We therefore turn to an alternative formulation in which worker's consumption depends on permanent income rather than spot earnings.

1.2 Efficient Contracts With Self-Insurance

In the previous analysis the employer's concern with compensation extends only to the first moment of the worker's consumption. Any compensation scheme that satisfies this average is as good as any other, and so this degree of freedom in \( y(\theta) \) was used to satisfy (4). We have already noted that (4) is the main content of the permanent income theory of consumption, so the distinguishing characteristic of the previous model is that any saving was done by the firm as the worker's agent. Now we assume that workers can perform these functions for themselves, so the analogy to the permanent income model is exact. In this case, however, the design of the optimal employment agreement must account for the utility maximizing consumption decisions of workers.
Consumption decisions are necessarily dynamic, and in a stochastic environment they involve expectations about the evolution of income and wealth in future periods. Nevertheless, when workers maximize expected lifetime utility subject to a lifecycle budget constraint, transitory shocks to current income will have only wealth effects on current consumption decisions. Bewley (1977), relied on the law of large numbers for this case to show that wealth effects become negligible as the period of income shocks shrinks relative to the length of the consumer's horizon. The impact of income risks on consumption is effectively diversified over a large number of periods. Consequently, for transitory income fluctuations that have only small effects on lifetime wealth, consumption decisions in any period are made as if they were constrained by expected rather than actual income. Given the distributions of employment and earnings, this implies that the worker's relevant budget constraint is that expected net savings must be nonnegative.

From a worker's perspective, a job is completely described by an arbitrary distribution of hours worked, \(\phi(h)\), and an arbitrary earnings function \(y(h)\). The worker possesses these two pieces of job-related information \textit{ex ante}, and he maximizes expected utility by choosing the consumption function \(c(h)\):

\[
(6) \quad \max \int u(c(h), T-h) \phi(h)dh \quad c(h)
\]

subject to the aforementioned budget constraint on net savings

\[
(7) \quad 0 = \int \{y(h) - c(h)\} \phi(h)dh.
\]

The Lagrangian for this problem is maximized pointwise for each \(h\), yielding

\[
u_1(c(h), T-h) = \lambda \psi h
\]

which is identical to (4). Thus, given \(y(h)\) and \(\phi(h)\) the worker's
privately optimal consumption policy equates the marginal utility of consumption in all states of nature. This marginal condition summarizes the worker's behavior, and it is a constraint on the contractual choice of employment and compensation strategies.

Given the density $\phi(h)$, the worker's preferences over earnings functions $y(h)$ extend only to the first moment in the sense that all earnings functions with the same mean must yield the same value of maximized utility in (6). In this sense, the worker behaves as if he is income risk neutral, given $\phi(h)$. It follows that if $\phi(h)$ is chosen to be the density on hours worked that is induced by the efficient solution to (1)-(2), say $\phi^*(h)$, and if $E_{\phi^*} y(h) = E_f y(\theta)$, the worker's private solution to (6) will yield the same consumption at each $h$ as in problem (1) and just attain $U^*$. Moreover, we are left with freedom to choose the shape of the earnings function since the shape of $y(h)$ is no longer determined by the requirements of reallocating risk. That is, $y(\theta)$ is no longer unique, though $c(\theta)$ is uniquely determined by the consumer's optimizing behavior.

We have already noted that condition (3) requires monitoring and enforcement to assure that it is satisfied. An alternative to explicit enforcement is to allow the firm complete freedom to choose employment by maximizing \textit{ex post} profits, yielding for each $\theta$

\begin{equation}
(\theta) \quad g_1(h; \theta) = y'(h),
\end{equation}

so $y'(h)$ is explicitly recognized as the firm's private (marginal) employment cost. From (3), efficiency requires $y'(h) = u_2/u_1$, which is the worker's value of time. The solution we propose is to view (8) as an additional constraint on the choice of a contract, and to choose the earnings schedule to satisfy efficiency. This additional constraint uniquely determines $y(h)$. 
At first the solution to this problem may appear trivial: choose the compensation schedule \( y(h) \) coincident with the reservation indifference curve \( U^* \) so that \( y'(h) \) traces out \( u_2/u_1 \). Such contracts are often proposed in the literature (e.g., Hall and Lilien, 1979) yet our discussion accompanying Figure 1 showed this schedule to be inefficient since the worker can generally achieve \( Eu(c,T-h) > U^* \) by efficiently timing consumption.

More generally, the self-enforcing earnings function that yields the efficient employment density \( \phi^*(h) \) and expected utility \( U^* \) must solve the following variational problem: choose \( y(h) \) such that \( y'(h) = u_2/u_1 \) and subject to the conditions that \( E(u(c,T-h)) = U^* \) and \( u_1 = \lambda \), as constant. Both \( u_2(c,T-h) \) and \( u_1(c,T-h) \) are defined from the efficiency condition and (4).

Omitting tedious details, the (unique) solution is

\[
(9) \quad y(h) = c(h) + \frac{U^*-u(c(h),T-h)}{\lambda}
\]

or, rewritten,

\[
(9') \quad U^* = u(c,(h),T-h) + \lambda(y(h)-c(h)).
\]

By taking expectations on both sides of (9) it is clear that \( Eu(.) = U^* \), and differentiation yields \( y'(h) = u_2/u_1 \). It is worth emphasizing that this solution for \( y(h) \) is unique only with respect to a particular employment density \( \phi^*(h) \): different distributions of employment will imply different compensation schedules. We develop this point below.

According to (9), optimal earnings exceed consumption (net saving is positive) when realized utility is less than expected utility, and conversely. The relationships among earnings \( y(h) \), consumption \( c(h) \), and expected utility are depicted in Figures 2a-c for alternative assumptions about the function form of \( u(.) \). We discuss Figure 2a where \( u_{12} > 0 \) and consumption declines with time worked. The important point is that the relevant values of
Figure 2
\( \frac{u_2}{u_1} \) that determine \( y'(h) \) are not measured along \( U^* \), but are determined by the slopes of indifference curves at the optimal consumption-leisure pairs along \( c(h) \). Since leisure must be normal in this case, for the same value of \( h \) these marginal rates of substitution exceed the rates along \( U^* \) when \( u(.) > U^* \), and are smaller when \( u(.) < U^* \). Thus \( y(h) \) is steeper than \( U^* \) to the right of \( T-h \) and flatter to the left. Finally, from (9), \( y(h) = c(h) \) when \( u(.) = U^* \), so \( y(h) \) is tangent to \( U^* \) from below. The same reasoning about the properties of \( y(h) \) applies when \( u_{12} < 0 \). Notice, by comparing Figure 2b and 2c, that normality of leisure is a necessary and sufficient condition for savings to rise with earnings. In fact \( y(h) \) and \( c(h) \) (and \( U^* \)) coincide only in the special case where the income elasticity of demand for leisure is globally zero. This is the class of utility functions

\[
(10) \quad u(c, T-h) = V(c+R(T-h))
\]

where \( R(T-h) \) is a concave function. Only in this case is the marginal utility of consumption fixed along an indifference curve, so the permanent income hypothesis happens to stabilize utility as well.\(^{17}\)

The efficient contract defined by the pair \( (y(h), \phi*(h)) \) possesses two desirable properties: it is self-enforcing and, as we will clarify, earnings are an increasing function of time employed no matter what functional form assumptions are made about the worker's utility. Nevertheless, Figures 2a-c indicate that optimal consumption-leisure pairs lie along successively higher indifference curves as hours decline if savings rise with income, which we assume. The indifference curves refer only to the utility associated with contemporaneous consumption and leisure and do not include the value of savings.

Consider again the worker's maximization problem in allocating consumption, where \( y(h) \) and \( \phi^*(h) \) are given. The value of state-h to the worker is the increment to expected utility when \( h \) becomes more likely. By
the Envelope Theorem this increment is the derivative of the Lagrangian defined by (6) and (7):

\[ \frac{dU^*}{d\phi^*(h)} = u(c(h), T-h) + u_1[y(h)-c(h)] = U^* \]

applying (9') for each \( h \). Therefore, the utility of goods and leisure consumed plus the utility-equivalent value of net saving is identically \( U^* \) in each state. Thus, (11) implies that the contract leaves the worker \textit{ex post} indifferent among states of demand, and \( y(h) \) is a full-utility reduced form indifference curve. It is a level set of the indirect utility function defined on income and leisure, given the known hours density \( \phi^*(h) \). The payment \( y(h) \) need not equal the actual earnings of a worker in a period, and it obviously need not equal the worker's consumption. What is required for efficiency is that \( y(h) \) be the cost to the firm of employing the worker for \( h \) hours. This cost may include both present and future compensation, so the connection among current compensation, labor cost, and consumption may be quite remote.\(^{18}\)

The derivative of \( y(h) \) defines the constant-marginal-utility-of-income labor supply schedule that is commonly used to characterize temporal variations in labor supply (Heckman, 1974; MaCurdy, 1981; Heckman and MaCurdy, 1980). These are also permanent income models. Employment variation in an efficient contract is determined by the same features of worker utility and intertemporal substitution that guide voluntary labor supply decisions over the lifecycle. This added margin of substitution eliminates wealth effects, so employment always rises with \( \theta \) and the response of employment along \( y(h) \) exceeds even the Slutsky-compensated labor supply response that would occur along \( U^* \) (see Figure 2).

This relation to voluntary labor supply raises two issues. First, given workers' ability to save, what distinguishes a contractual equilibrium from an
auction market in which workers make voluntary labor supply decisions at a
competitive (state-dependent) wage? Second, if the market is organized around
contracts as described here, what can be learned from labor supply estimates
that assume unconstrained choices by workers? We return to these questions
below.

2. Further Properties of Efficient Contracts

2.1 Duality

We have shown that when the earnings function satisfies the problem set
out above, the worker is ex post indifferent among realizations from the den-
sity \( \phi(h) \). Suppose we reverse matters and permit both consumption and hours
choices to be made by workers. The analogy to the earlier problem is that the
worker, presented with an arbitrary compensation schedule \( y(h) \), chooses the
fraction of each year (or comparable unit of time) spent at employment level
\( h \). Thus, the problem is to choose the pair of functions \( c(h) \) and \( \phi(h) \)
subject to the budget constraint and the "time" budget \( 1 = \int \phi(h)dh \):

\[
(13) \quad \max \int u(c(h),T-h) \phi(h)dh + \lambda \int [y(h)-c(h)] \phi(h)dh + \delta [1-\int \phi(h)dh].
\]

where \( \delta \) is a Lagrange multiplier. We maximize pointwise on \( h \), so the
first-order conditions are \( u_1 = \lambda \) and, for \( \phi(h) \),

\[
(14) \quad \delta = u(c(h),T-h) + \lambda (y(h)-c(h)).
\]

Since \( \delta \) is constant, (14) implies that proportions of time will be allocated
so as to equate incremental full utility at each hours level where \( \phi(h) > 0 \).
By differentiation, this allocation of time and consumption also implies that
\( y'(h) = u_2/u_1 \) as above. Finally, since \( \phi(h) \) is a density, taking expecta-
tions in (14) implies \( \delta = Eu(c(h),T-h) \), which is the maximized value of
expected utility.
We have already seen that the firm optimizing against (9) will choose an hours worked density in which the worker is also indifferent among states of labor demand. The difference between these two problems is that the firm's choice that leads to an hours distribution is unique, but the worker's is not. For example, if \( y(h) \) is given by (9), Figure 2a indicates that an equally attractive solution to \( \phi^*(h) \) locates point mass at \( \bar{h} \), yielding \( U^* \) with certainty. The worker is indifferent between spending all his time at \( \bar{h} \) and the firm's choice \( \phi^*(h) \). Since \( \bar{h} \) is a degenerate density, the worker is also indifferent among mixtures of \( \bar{h} \) and \( \phi^*(h) \) when compensation follows \( y(h) \). In contrast, if \( y(h) \) were coincident with \( U^* \) the worker would never specialize at one \( h \), since the ability to consume out of permanent income guarantees \( \text{Eu}(\cdot) > U^* \) when \( \phi(h) \) has dispersion.

The family of densities that are utility-equivalent is easily characterized. Let \( \phi^*(h) \) be a density that satisfies the necessary conditions. From (14) we know that any transformation of \( \phi^*(h) \) that yields the same utility must leave \( c(h) \) and \( \lambda \) unchanged. Consider the function \( \mu(h) \) with the properties that \( \phi^*(h) + \mu(h) \geq 0 \) for each \( h \) and \( \int \mu(h)dh = 0 \). Thus \( \phi^* + \mu \) is a density function. Applying (14)

\[
0 = \int \left\{ u(c(h),T-h) + \lambda(y(h)-c(h)) \right\} \mu(h)dh.
\]

(15)

Note that (15) holds for any \( \mu(h) \). If, in addition,

\[
0 = \int \{y(h)-c(h)\} \mu(h)dh,
\]

(16)

then equation (15) implies that

\[
0 = \int u(c(h),T-h) \mu(h)dh.
\]

Therefore, any density that balances the budget equation for consumption function \( c(h) \) is a solution to the utility maximization problem. Consequently, a given earnings function may be associated with an infinity of employment
distributions that are equally attractive to the worker.

This argument also implies that the only transformations of \( \phi'(h) \) that can raise expected utility are ones that produce negative expected net saving. This violates the budget constraint, so that the necessary conditions clearly define a maximum. This, in turn, is proof that the firm's choice of a density lies within the worker's preferred set. This does not mean that given \( y(h) \), the worker and firm will make the same choices. The firm's choice of \( y(h) \) is not uniquely optimal for the worker. Even if it were, the worker has no incentive to condition the timing of \( h \) on \( \theta \), although the firm clearly has. What can be said is that it is costless for the worker to delegate hours decisions to the firm.\(^{19}\) Conversely, any change in the employment density that causes expected net saving to rise for a fixed \( y(h) \) must reduce expected utility. To accept this change, workers would have to be compensated via a shift in the function \( y(h) \) that brings expected utility back to \( U^* \).

The characteristics of equilibrium compensation are determined by worker attitudes toward employment risk, to which we now turn.

2.2 Attitudes Toward Employment Risk

A fundamental aspect of contract-insurance models is that income claims are assumed to be tradable across states, while claims to time — as a tied good — are not. Workers who achieve perfect consumption insurance must still hold employment risks that are specific to an individual employer or occupation. In the equilibrium of our model, the efficient levels of compensation and employment variation are jointly determined by technology and demand conditions — the distribution of \( \theta \) — and by the willingness of workers to hold the employment risks associated with a given job.

A careful distinction is warranted between wage uncertainty and employment uncertainty. Risks in consumption are fully insured in this
analysis. If uncertainty is the result of stochastic wages and workers can choose their hours at each realized wage, then a riskier distribution of wages is preferred. This is because there are no income effects with $u_1 = \lambda$, and a mean-preserving spread in wages affords the opportunity to work more in high wage states and less in low wage ones, raising average income.\textsuperscript{20} Indirect utility is convex in wages. In contrast, since Smith's discussion of the market for Scottish bricklayers,\textsuperscript{21} economists have been concerned with equalizing differences for increases in employment risk. In a contractual market the distribution of $h$ is a primary characteristic of a job, meaning that the latter notion of risk is appropriate. In general, increases in this type of risk will not be preferred.

Consider the shape of the earnings functions in Figures 2a-c. These functions resemble indifference curves, and in fact they are the level sets of full-utility defined on income and hours. They are flattened relative to $U^*$ by the ability to save. Since $y'(h) = u_2/u_1$ the curvature of $y(h)$ is determined by

$$y''(h) = \frac{u_{12}^2 - u_{11}u_{22}}{u_1u_{11}} = -\frac{D}{u_1u_{11}}$$

where $D$ is the Hessian determinant of the utility function. If utility is strictly concave ($u_{11}, u_{22} < 0; \ D > 0$) then the efficient earnings function is convex, as illustrated, and the implicit supply price of time is increasing in an optimal contract. In contrast, if utility is homogeneous of degree one both $y(h)$ and $c(h)$ are linear and the supply price is constant. Since $u_1$ is fixed after consumption decisions have been optimized, these cases correspond to rising and constant marginal disutility of work, respectively. Therefore, we have the plausible result that the degree of employment risk aversion in workers' preferences is inversely indexed by the elasticity of the
implicit labor supply schedule, \( y'(h) \), in the optimal contract. If labor supply is perfectly elastic \( (y''(h) = 0) \) then workers are employment risk neutral. Notice that preference for employment risk \( (y''(h) < 0) \) is not precluded by our analysis, though we assume both convex indifference curves and aversion to consumption and leisure risks individually \( (u_{11} < 0; u_{22} < 0) \).

To illustrate the connection between \( y(h) \) and preferences, consider a small change in the shape of the employment density, satisfying \( \int d\phi(h)dh = 0 \). By the Envelope Theorem, the effect of this transformation on expected utility is

\[
dU^* = \int \{u(\cdot)+u_1(y-c)\} d\phi(h) \, dh
\]

\[
= U^* \int d\phi(h)dh = 0
\]

when \( h(h) \) is defined by (9). Thus, for example, a small change in the riskiness of \( \phi^*(h) \) has no impact on expected utility, independent of the properties of \( u(\cdot) \). In contrast, if earnings were redistributed across states so as to be lower than the optimum in the tails of the density, then one may verify that a change in \( \phi(h) \) that increases the mass in those tails would reduce expected utility. Thus, (18) does not imply risk indifference. Rather, the special feature of \( y(h) \) is that it incorporates attitudes toward risk, yielding exact utility-compensating changes in expected earnings for small changes in the density \( \phi^*(h) \).

This interpretation of \( y(h) \) as expected utility compensating also holds for discrete changes in the employment distribution that satisfy equation (16). For those that do not, we know by our arguments above that the efficient compensation schedule must shift if expected utility is to be maintained at \( U^* \). In this case, the displacement of the first-order conditions reveals that \( dc(h)u_{11} = d\lambda \) for each \( h \) and

\[
dy(h) = -\frac{d\lambda}{\lambda} [h(h)-c(h)].
\]
Therefore, if (16) is not satisfied, changes in the distribution of employment cause rotations in \( y(h) \) along the indifference curve \( U^* \).

Consider an increase in employment risk that is generated by a mean preserving spread in the density \( \phi^*(h) \). This change in risk will satisfy condition (16) if the savings function \( s(h) = y(h) - c(h) \) is linear. In this case, \( y(h) \) is exactly utility compensating: with risk aversion the convexity of \( y(h) \) implies that average earnings and consumption rise to maintain \( U^* \) — applying Jensen's inequality — but consumption in each state of nature remains unchanged.\(^{22}\) Conversely, if \( s(h) \) is convex, (15) and (16) imply that an increase in risk causes expected utility to fall for a fixed \( y(h) \). To maintain \( U^* \), a consumption premium must be offered in each state of nature and, applying (19), the optimal earnings function rotates clockwise along \( U^* \) (when leisure is a normal good).\(^{23}\) We therefore have a second sense of aversion to employment risk: workers demand a consumption premium in each state of nature for bearing risk if the savings function is convex in time worked. Equivalently, this condition implies from (9) that

\[
(20) \quad \frac{d^2u(c(h),T-h)}{dh^2} < 0.
\]

Workers are averse to employment risks if, and only if, post utility in the optimal contract is concave in hours.

Note that (20) is stated in terms of the indirect utility that a worker receives from \( h \), consumption decisions having been optimized. Because the curvature of \( c(h) \) depends on third derivatives of \( u(.) \), however, it is difficult to make a general statement about the class of direct utility functions that will satisfy (20).\(^{24}\) We therefore take (20) as a definition of worker aversion to employment risk. Consequently, when workers are risk averse in the sense of (20) an increase in employment risk will cause \( y(h) \)
Fig. 3 - An Increase in Employment Risk when Leisure is a Normal Good
to rotate along $U^*$, while the entire consumption profile shifts upward (see Figure 3). Therefore, although indifference curve $U^*$ is never an efficient compensation schedule, it is the envelope of all possible $y(h)$ functions associated with all employment densities $\phi(h)$. With identical workers, differences in $\phi(h)$ across jobs generate this family of payment schedules in market equilibrium.25

2.3 Implications for Empirical Studies of Labor Supply

The theoretical literature on employment contracts is clearly inconsistent with empirical models used in the literature of labor supply. If one were to adhere strictly to the full risk-shifting hypothesis, the conclusion must be that what pass for studies of labor supply are instead studies of consumption behavior. That is, instead of estimating the function $y(h)$ (or its derivative) in Figure 2, the function $c(h)$ is estimated and called a supply function. This dichotomy of ideas is the more striking in light of the popularity of both literatures among labor economists.

Of course, we doubt that compensation follows the consumption schedule. Rather, the implicit labor supply function we characterize, which determines equilibrium employment variation, conforms to models of the lifecycle allocation of time and intertemporal substitution in consumption decisions (e.g., Lucas and Rapping, 1970). Even so, the link between the contracts that we anticipate and available estimates of labor supply is remote. To this point, we assume a compensation schedule of the form $y = y(h, \phi, U^*)$, and so the hours response to changes in labor's marginal product is measured as $y''(h)^{-1}$, which describes motion along the constant marginal utility of income function. Assuming for the moment that the marginal wage is properly measured empirically, this same response is implied in lifecycle models as discussed by Ghez and Becker (1975) and Heckman and MacCurdy (1980), among others. An
important difference is that the supply function implied by our analysis depends on the employment density, $\psi(h)$. If a job implies a unique density, as we assume, then the implication is that the labor supply schedule shifts with changes in jobs. If expected utility is unaltered by these moves, then shifts in labor supply amount to rotations about $U^*$. Consequently, life cycle studies that ignore job changes mix the more elastic responses along constant marginal utility schedules with the less elastic responses along the indifference curve $U^*$. Responses of the second type are the ordinary compensated ones with which most of the labor supply literature has been concerned.

Unfortunately, the theoretical framework underlying empirical labor supply studies assumes an institutional setting in which firms announce wage offers and workers choose the number of hours they will work. Yet we are unaware of important occurrences that can be characterized according to this arrangement. Firms either make tied wage-hours offers or rules of compensation are mutually agreed to ex ante and firms make ex post hours choices. In all but the most remote cases, this means that variations in average earnings are a poor representation of variations in marginal product.

At the risk of oversimplification, assume an efficient contract of the type characterized above. Assume that $y(h)$ is observed, that is, no component of compensation is deferred. Assume further that an empirical study first computes an average wage, $w = y/h$ and then infers wage responses as $(dw/dh)$, where

$$\frac{dw}{dh} = \frac{1}{h} \left[ y'(h) - \frac{y(h)}{h} \right].$$

Such studies literally estimate hours response from the average rather than the marginal compensation schedule. Thus, while estimates of the $y/h$ function presumably imply the short run response, $y'(h)$, it is clear that the available literature does not distinguish the two functions. Instead, the
bulk of the estimates refer to the average wage schedule as if they were estimating a Slutsky-compensated schedule or a marginal-utility-of-income-constant labor supply function. A related point is that most labor supply studies are based on averages calculated over some period such as a year, thus concealing considerable intraperiod variation. In each of these cases, the measured wage-hours combination will not lie on the compensation schedule unless it is linear, which obtains only with risk indifference and perfectly elastic supply. Even in this case, if \( y(0) > 0 \) then \( y'(h) \neq y(h)/h \) and average wages do not measure the shadow price of time. As such, in a contractual setting, the estimates derived by these procedures are uninterpretable.

2.4 Employment and Earnings Without Contracts

Nothing in our analysis contradicts the validity of the classic classroom example of a spot market equilibrium where workers choose their hours and firms choose their employees. The conditions required for this kind of market are that (i) labor is freely mobile among firms, (ii) there are no firm-specific skills, and (iii) in production each hour worked substitutes one-for-one for every hour.\(^26\) The final condition — that effort acquired at the intensive margin of more hours per worker is as productive as hours acquired at the extensive margin of more workers — insures equality of average and marginal wages in equilibrium. In this market, a worker's income is the product of the market wage and his chosen hours worked.

Unlike the contract case where workers are indifferent among states of labor demand, in a spot market they will prefer realizations of demand that yield higher hours worked. Assume that the market-wide demand for labor depends on a random variable \( \theta \). As before, assume that workers efficiently distribute their incomes across states so that consumption, \( c(h) \), follows a constant marginal utility of income path. Associated with \( c(h) \) is a labor
supply schedule, \( y'(h) = u_2(c(h),T-h)/u_1 \), and a full-utility indifference curve, \( y(h) \), that yield expected utility \( U^* \) to workers in this market. As above, the position of \( y(h) \) and the labor supply function depend on the equilibrium distribution of \( h(\theta) \). Spot market income is not equal to \( y(h) \), however. Given the labor supply schedule, \( y'(h) \), and a fixed number of workers, the equilibrium average hourly wage in demand state \( \theta \) is \( w(\theta) = y'(h(\theta)) \), and so earnings are \( y'(h(\theta))h(\theta) \). For any \( h \) the full utility that workers receive is \( u(c(h),T-h) + u_1(y'(h)h - c(h)) \), as above. This function is increasing in \( h \) for \( y'(h) > 0 \), so states of demand that require greater hours are preferred if \( y(h) \) is convex. From (17), this requires that \( u(c,T-h) \) be strictly concave.

The condition that \( y(h) \) is convex implies that spot market earnings will exceed contractual earnings along \( y(h) \) when hours worked are relatively high, and conversely when hours are low. Since the expected value of earnings must be the same under the two regimes, we conclude that the variance of earnings is greater in a spot market, holding expected utility and the optimal distribution of hours fixed. This reduction in earnings variance induced by a contract is, of course, unrelated to any risk shifting motives, since reductions in earnings variance have no direct value to workers.

3. **Conclusions: Taking Stock**

We began by noting the widespread, and well-known, durability of employment relations. There are many plausible reasons for this durability; specific capital or simple mobility costs are prominent candidates. Even without these, however, reference to Figure 1 and the surrounding discussion shows that when there is distribution of available levels of employment, \( h \), in a market, there are economies through consumption smoothing of having each worker experience them all rather than relying on low-\( h \) and high-\( h \)
specialists. Yet, when mobility costs and firm-specific demand variation requires that employment vary with a durable job, we have argued that employment contracts (either explicit or implicit) are necessary. A prime characteristic of these contracts is that workers willingly and efficiently delegate the choice of the level of employment to firms.

We obviously do not believe that risk-shifting is an important force generating durable relations, or even that it is a significant motive in employment contracts that are made necessary by training specificities. We know of no important features of employment agreements, or labor markets generally, that require shifting of income risk as an explanation. Yet, the assumption that workers live from hand-to-mouth while receiving income insurance from employers predominates in recent literature. Here, without dropping the plausible assumption of risk aversion, we allowed workers' consumption decisions to be guided by the more sensible constraint of lifetime wealth rather than current income. Under general conditions, contracts that account for the ability to save are efficient, and workers are indifferent among realizations of labor demand. This finding does not depend on ad hoc assumptions about the functional form of worker utility. Nevertheless, attitudes toward employment risk are well defined, and differences in distributions of labor demand among jobs generate an equilibrium distribution of labor cost schedules. Riskier jobs offer an earnings premium, even with perfect capital markets.

The added, intertemporal, margin of substitution allowed by consumption smoothing implies that labor supply in efficient labor contracts is correspondingly more sensitive to transitory fluctuations in labor's marginal product than in a market with no savings opportunities. Operationally, however, efficient contracts also break the connection between labor's average
rate of compensation — the "wage" in all studies of labor supply — and the marginal productivity shocks that drive contractual changes in employment along the shadow supply schedule. Thus, the relation between labor supply elasticities estimated by conventional methods and their "true" counterparts is at best remote. This fact may help to resolve the well-known paradox of estimated labor supply elasticities based on individual data: they are simply too small to rationalize observed fluctuations in aggregate employment.\textsuperscript{27} In our analysis, however, the structural parameters of labor supply and utility determine the market equilibrium distribution of earnings and employment variability among jobs, as in Section 2.2. As should be clear from that discussion, it is generally feasible to recover the true parameters of implicit labor supply from this distribution of equalizing differences for employment variation. A full empirical implementation of this point is an important extension of the current analysis.
FOOTNOTES

1 See the evidence presented in Akerlof and Main (1981), Hall (1982) and Oi (1983), on the duration of jobs.


3 On this point, see also Hall and Lilien (1979).

4 We use the term compensation schedule to mean total payment for a specified number of hours worked. The wage rate, in contrast, is compensation per hour.

5 It is possible for an optimal contract to require negative severance pay. We develop this point in Section 2.3.

6 Rosen's (1983) survey is a good example. There the term "implicit contract" is virtually synonymous with contracts that share risks and provide insurance to workers.

7 In particular, see Hall (1978), Hall and Mishkin (1982), and Flavin (1981). On the consumption behavior of the unemployed see Hamermesh (1982), Burgess and Kingston (1981), or M. Grossman (1973).

8 While we are not aware of important third-party income insurance schemes, other than unemployment insurance, we frankly wonder whether firms have comparative advantage in offering such insurance. They clearly do not in the horizontal sense of risk pooling simply because the product demand shocks that affect a firm's profitability translate directly into labor demand shocks that affect the worker's income. This positive covariance between worker income and firm profit severely limits horizontal pooling.
An optimal payment schedule must have at least one point in common with $U^*$. Increasing or decreasing hours from this point must cost less than along $U^*$ when income is transferable across states and the marginal utility of consumption is not constant.

An additional assumption underlying (4) is that shocks to labor demand have no wealth effects on the optimal consumption policy of the worker. The limiting conditions needed for wealth effects to vanish are spelled out in footnote 14, below, and by Bewley. If these simplifying conditions are not satisfied, a fully dynamic analysis is required, which is beyond the scope of this paper.

Given non-separability of consumption and labor supply behavior, empirical tests of the permanent income hypothesis that rely on the stochastic behavior of consumption, as in Hall (1978), are clearly incorrect. The optimal covariance between transitory changes in income and consumption can be positive or negative. The unobservable marginal utility of consumption should follow a random walk.

Reductions in employment are preferred in the additional sense that a shift in the distribution of hours in favor of low demand states of nature would increase expected utility.

Green and Kahn (1983) study "second-best" risk shifting contracts in which only the employer observes $\theta$. The tradeoff between efficient choices of $h$ by the firm and efficient risk pooling leads to a payment schedule that is increasing in $h$. Nevertheless, they show that employment reductions are still preferred by workers when leisure is a normal good.

Bewley assumes that workers may save but not borrow and that income follows a stationary stochastic process. He proves the following. Let $\beta < 1$ be the discount factor and let $T$ be the consumers planning horizon.
Denoting the marginal utility of consumption by \( \lambda_t \) in period \( t \), as \( t \to \infty \), \( \beta + 1 \), and \( T \to \infty \), it must be that \( \lambda_t + \lambda \), a constant, almost surely. Thus, even without borrowing the wealth effects of transitory income disturbances will be small, so the marginal utility of consumption is approximately constant. On these points, see also Schectman (1973, 1976) and Yaari (1976).


16 An implicit assumption is that shocks must be independent, otherwise the expected future hours density will be conditioned by past observations and conditional expectations may deviate from those necessary to sustain the contract. Although the existence of serially correlated disturbances is a source of instability, the existence of specificities and explicit mobility costs serve as a counterbalance or stabilizing force.

17 This is the utility function employed in Grossman and Hart (1981), Rosen (1983), and Hall and Lilien (1979). This choice is felicitous, since it eliminates saving (by either party) as a valid concern in constructing a contract, and yields a utility guarantee for the worker.

18 Nothing in our analysis guarantees that \( y(h) \) is positive over its entire range. For example, if \( U^* = u(0, T) \), so workers are indifferent between participation and non-market alternatives, then \( y(h) \) is negative over some range of \( h > 0 \). Thus if \( U^* \) is sufficiently low, the efficient payment schedule requires payments by workers to firms at low \( h \). In a contract with continuous trade, this low hours tax may occur through reductions in other forms of (deferred compensation), though if these are not feasible the constraint \( y(h) > 0 \) is an additional constraint on the problem. We are indebted to a referee for this point.
This is an artifact of the presumption of static worker utility. If worker preferences were also variable, e.g., shocked by contingencies like illness and the availability of leisure complements, such as the presence of other family members, then the worker's choices would be unique and a joint profit/utility maximization would be impossible in the sense of first-best optima. For a related point, see Hall and Lazear (1984).

The point that price variability is preferred originates with Oi (1961), who noted the convexity of the firms profit function in price. The logic is that: with greater wage variation the worker always has the opportunity to leave his hours unchanged, so average income and utility would remain fixed. A small reallocation of hours from low to high wage states will raise average income, and must be a utility improvement.

Smith (1977) Book 1, Chapter 10.

The linearity of \( s(h) \) implies (16) for a mean preserving spread, and the change in expected utility at the original levels of consumption is

\[
\int u(c(h), T-h) \mu(h) dh = 0, \text{ applying (15).}
\]

When leisure is inferior, \( y(h) \) rotates counterclockwise along \( U^* \). Reference to Figure 2c shows that consumption rises in each state of nature in this case as well.

The class of linear homogeneous functions yields risk neutrality, but even concave transformation of these need not satisfy equation (19).

A further implication of this fact is that hours variation within a job is compensated differently from hours differences across jobs. For example, when leisure is normal, transitory increases in hours are less costly to the firm than permanent ones.

This is a generalization of the one firm-one worker technology analyzed above. If that technology is maintained the main conclusions of this section
continue to hold, though the analysis is more complex. Specifically, with market-wide randomness in the demand for labor, increases in hours are preferred. A key difference is that if productivity shocks, \( \theta \), are imperfectly correlated across firms there will be no unique market wage since optimal hours differ among firms. Equilibrium is supported by a hedonic surface, \( y(h) \), relating hours and compensation. If the \( \theta \) are i.i.d. among firms, then \( y(h) \) is unique and expected utility is \( U^* \). If shocks are correlated among firms, then \( y(h) \) depends on market demand in a period, though saving activity implies the equilibrium \( y(h) \) functions are a family of vertically parallel indifference curves. Because greater aggregate demand shifts \( y(h) \) up, high demand states are preferred by workers.

For a summary of the evidence, see Altonji (1982) and the references cited there.
REFERENCES


, "Heterogeneous Firms and the Organization of Production," Economic Inquiry, XXI (April 1983).


