SECRECY OF MONETARY POLICY AND THE VARIABILITY OF INTEREST RATES

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Abstract

This paper addresses the issue of how secrecy of the short run monetary policy objectives affects the behavior of the federal funds rate. Secrecy is modelled by assuming that financial markets are uncertain about a parameter in the Federal Reserve reaction function. They learn over time about this parameter, by means of Bayes rule; this learning process is reflected in the time path of interest rates and of reserve aggregates. The main result of the paper is that secrecy tends to increase the volatility of the funds rate and of reserve aggregates.
1. Introduction

The short term objectives of U.S. monetary policy are deliberately kept secret by the Federal Reserve System until after they have become obsolete. Specifically, the short-run money targets and the tolerance ranges for fluctuations in the federal funds rate, chosen at each monthly Federal Open Market Committee (FOMC) meeting, are revealed to the public two weeks after the next FOMC meeting has set new targets.

This secrecy has been justified by the Federal Reserve on a number of grounds, the main one being that public disclosure of current monetary policy decisions would increase the variability of interest rates. According to Goodfriend (1986), it is precisely this point which allowed the FOMC to win a legal case in which public disclosure was at issue: by claiming that public disclosure would raise the variability of interest rates, the FOMC argued that it would also raise the level of interest rates (by increasing the risk premium), and that this would harm the commercial interests of the Treasury.

The validity of this and other arguments against public disclosure has recently been questioned by a number of authors (see Brunner 1981, Goodfriend 1986). The purpose of this paper is to elucidate some of the issues involved in this debate, by analyzing how secrecy of the Federal Reserve policy targets affects the equilibrium behavior of short term interest rates in a simple theoretical model of the market for bank reserves. The focus of the paper is on whether secrecy makes monetary control more or less difficult in the period between one FOMC meeting and the next. Thus, the broader question of what are the costs that secrecy imposes on the private sector, in terms of wrong economic decisions and in terms of real resources wasted in "Fed watching" activities, is neglected.
The central point of the paper is that financial markets react to the secrecy of the FOMC meetings by trying to learn about the undisclosed policy objectives. Knowing the structure of the Federal Reserve decision process, market participants can gradually learn about the secret policy targets from their observations of the federal funds rate and of other variables influenced by the action of the Federal Reserve. This learning process is described in the paper by means of Bayes rule. It is shown that market beliefs about the true policy target have to be included in the set of fundamentals to which interest rates react. The main result of the paper is that the market learning process tends to increase the unconditional variance of short term interest rates and of reserve aggregates, as well as their contemporaneous reaction to temporary shocks to the market for reserves. This result contradicts one of the points raised by the Federal Reserve against public disclosure, and indicates that secrecy can be detrimental to the achievement of the Federal Reserve objectives.

The result that keeping the market uninformed increases the volatility of asset prices is far from obvious. On the contrary, it contrasts sharply with the existing literature on the specific topic of this paper (Dotsey (1985) and Rudin (1986)), and on the volatility of asset prices in general (Shiller (1979), LeRoy and Porter (1981)). In this literature, lack of information is generally modelled by means of an additive random term in the linear stochastic process driving the market fundamentals -- e.g., the policy variables. This additional random term obfuscates the signal extraction problem solved by financial markets and tends to reduce the reaction of asset prices to the arrival of new information. Through this channel, additional uncertainty generally reduces the unconditional variance of asset prices. In the model of this paper, instead, lack of information is
specified as parameter uncertainty (i.e., as uncertainty about some temporarily stable features of the exogenous stochastic process). Hence the uninformed agents have an opportunity to learn over time. Their learning process is the source of the additional volatility in asset prices. This finding is more general than the model within which it is derived, and can provide some new insights on how to explain the seemingly "excessive" volatility of asset prices in other contexts as well -- see also Tabellini (1986).

The paper outline is as follows. Section 2 describes the basic model of the market for reserves. Section 3 computes a closed form expression for the equilibrium value of the federal funds rate. Section 4 describes the market learning process. The main results of the paper concerning the consequences of secrecy are discussed in Section 5. The conclusions are summarized in Section 6.

2. The Model

The model is a very simple description of the market for bank reserves. The supply of total reserves, TRₜ, consists of borrowed (Bₜ) and non-borrowed (Nₜ) reserves:

\[ TRₜ = Bₜ + Nₜ \]  \hspace{1cm} (1)

The borrowing behavior of banks is approximated by:

\[ Bₜ = δ₀ + δ₁Fₜ + δ₂Fₜ₊₁/t + δ₃B₋₁ \]  \hspace{1cm} (2)

\[ δ₁ > 0, \quad δ₂, δ₃ < 0 \]

where \( Fₜ \) is the federal funds rate at time \( t \) and \( Fₜ₊₁/t \) is the funds rate expected for time \( t+1 \), based on the information available at time \( t \). The specification of equation (2) is motivated by the following
considerations. Under the current operating procedures, the marginal cost of borrowing from the discount window is the sum of two components: the discount rate (here assumed constant and implicit in the intercept term), and a non-price cost which increases over time, intended to dissuade banks from borrowing on a permanent basis. Thus, the higher is the borrowing done in the past, the higher is the marginal cost of borrowing again today, and hence the smaller is \( B_t(\delta_3 < 0) \). The marginal benefit of borrowing from the discount window is the federal funds rate (what the bank would otherwise have to pay on the federal funds market). An increase in \( F_t \), thus, induces the bank to borrow more today (\( \delta_1 > 0 \)). Similarly, if \( F_{t+1}/t \) increases, the bank expects that it will be borrowing more in period \( t+1 \); because of the increasing non-price marginal cost of borrowing repeatedly, the bank will cut back on its current borrowing (\( \delta_2 < 0 \)).

Non-borrowed reserves are determined by the Federal Reserve System according to the following reaction function:

\[
N_t = N^T + \gamma F_t + u_t, \quad u_t \sim N(0, \sigma_u^2), \quad \gamma > 0
\]

where \( u_t \) is a random shock and \( N^T \) denotes a target chosen by the FOMC and for simplicity assumed to be constant. Implicitly, it is assumed that the Federal Reserve also cares about interest rates: if \( F_t \) rises, non-borrowed reserves will be allowed to expand above target (\( \gamma > 0 \)).

Finally, total reserves demanded are given by:

\[
TR^d_t = \alpha_0 + \alpha_1 F_t + e_t, \quad e_t \sim N(0, \sigma_e^2), \quad \alpha_1 < 0
\]

where \( e_t \) is a random shock. An increase in the federal funds rate is supposed to reduce the demand for total reserves, either directly, through its impact on the demand for excess reserves, or indirectly, by leading to a reduction in the demand for bank deposits. Since the analysis refers only
to the very short run, nominal income and other variables entering the
demand for bank deposits are taken to be exogenous and can be thought of as
being incorporated either in the intercept or in the random term. To simpl-
ify the computations, the random variables $u_t$ and $e_t$ are assumed to be
mutually and serially uncorrelated. Each of these equations could be
modified in several ways, by adding more complicated dynamics, more expecta-
tion terms and more serially correlated random shocks, without changing the
nature of the results in any respect.

The value of the federal funds rate which clears the market for
reserves is:

$$F_t = \frac{1}{\gamma + \delta_1 - \alpha_1} \left( \alpha_0 - \delta_0 N^T + B_{t-1} \delta_2 F_{t+1/t} + u_t + e_t \right)$$

(5)

Thus, through the borrowing equation, the current equilibrium value of
the federal funds rate depends on the funds rate expected to prevail in the
future. The existence of an equilibrium relationship between $F_t$ and
$F_{t+1/t}$ is one of the two formal ingredients necessary and sufficient to
generate the results of the paper -- the second ingredient, discussed in the
following sections, is that secrecy gives rise to parameter learning. Note
however that the particular way in which the dependency of $F_t$ on $F_{t+1/t}$
is derived is quite irrelevant. Postulating a monetary policy of targeting
borrowed reserves, rather than non-borrowed reserves, would still generate
the same results which are presented below, since such a policy would give
rise to a semi-reduced form analogous to (5), in which expectations enter as
a determinant of $F_t$. By contrast, a policy of total reserve targeting
could remove the demand for borrowed reserves from the model. It would,
therefore, cut the link between current and expected future values of the
funds rate that drives the results. However, this link could be easily
reintroduced on the demand side of the market for reserves -- for instance by postulating that money demand is affected by interest rates of longer maturity than the funds rate itself, such as in Roley and Walsh (1985) or in Nichols, Small and Weber (1982). The results of the paper would then survive this modification, subject to some minor qualifications.

3. The Federal Funds Rate and the Market Beliefs

In this section, the closed form solution of the equilibrium condition in the market for reserves, equation (5), will be computed. These computations essentially amount to finding an expression for the expectation variable \( F_{t+1/t} \) in terms of variables determined at time \( t \) or earlier. In order to do that, the information set available to private agents at time \( t \) must be carefully specified.

Under secrecy, some or all of the terms on the right hand side of the policy reaction function, equation (3), are unknown to market participants. Throughout the paper it is assumed that the market knows the parameter \( \gamma \) on the right hand side of (3) and can observe the funds rate, \( F_t \), but it ignores the true value of the non-borrowed reserves target, \( N^T \). Moreover, it is also assumed that the random shocks \( u_t \) and \( e_t \), and the value taken by both total and non-borrowed reserves, \( TR_t \) and \( N_t \), are not observed by market participants until after \( N^T \) is revealed to them.

Of these hypothesis, the only crucial one is that secrecy gives rise to some parameter uncertainty -- here, to uncertainty about the parameter \( N^T \) in the policy reaction function. This form of uncertainty provides the market with an opportunity to learn about a (temporarily) stable feature of the policymaker behavior. And it is the existence of a learning process that generates the central results of the paper. The remaining hypotheses
can be relaxed without changing the nature of the results. More specifically, the hypothesis that \(N_T\), \(TR_T\) and either \(u_t\) or \(e_t\) are not observable enables the Federal Reserve to maintain some private information about \(N_T\): if it was violated, then private agents could infer the true value of \(N_T\) right away, by inverting equations (3) and (5). It could be relaxed, at the price of some minor formal complications, by adding more unknown terms to the right hand side of the policy reaction function, (2). Ultimately, therefore, this hypothesis can be interpreted as saying that the Federal Reserve has some random private information about the market for reserves, and that this information becomes available to market participants only with a time lag. Finally, the hypothesis that neither \(e_t\) nor \(u_t\) are observable serves the only purpose of simplifying the notation, and can be relaxed without complicating the formal analysis in any respect.

On the basis of such an information set and of its prior beliefs about the policy target, \(N_T\), the market will use (5) to form an optimal forecast of the funds rate next period. Specifically, by advancing equation (5) by one period, and then taking expectations conditional on the information set available at time \(t\), we obtain:

\[
F_{t+1/t} = \frac{1}{\gamma + \delta_1 - \alpha_1} (\alpha_0 - \delta_0 n_t - \delta_2 B_{t+2/t} - \delta_2 F_{t+2/t})
\]  
(6)

where \(B_t\) is given by equation (2) and where \(n_t\) is the expected value of \(N_T\), based on the market posterior beliefs updated with the information collected up to and including period \(t\). An expression for \(n_t\), obtained through Bayes rule, will be given in the next section.

By iterating equation (6) forward, or equivalently by using the method of undetermined coefficients, we obtain the following closed form solution for the equilibrium value of \(F_t\):
\[ F_t = \pi_0 + \pi_1 N^T + \pi_2 B_{t-1} + \pi_3 (e_t + u_t) + \pi_4 n_t \]  

(7)

Expressions for the \( \pi_i \) coefficients are obtained in the Appendix. There it is shown that: \( \pi_1, \pi_4 < 0 \), \( \pi_2, \pi_3 > 0 \), and \( \pi_1 = -\pi_3 \). These signs conform to intuition. They say that a shock to the demand for total reserves increases the equilibrium value of the funds rate \((\pi_3 > 0)\); a higher actual or expected target for non-borrowed reserves reduces it \((\pi_1, \pi_4 < 0)\); and a higher amount of borrowing in the previous period, by reducing the current supply of borrowed reserves, also increases the funds rate \((\pi_2 > 0)\).

Equation (7) incorporates the central result of the paper, that will be exploited more fully in the following sections. As a consequence of the secrecy concerning the FOMC decisions, market beliefs about the short run policy target, summarized by the variable \( n_t \), belong to the set of market fundamentals to which the federal funds rate reacts. The inclusion of \( n_t \) on the right hand side of (7) has an important implication: since \( n_t \) is a transformation of the underlying unobservable shocks to the market for reserves, \( e_t \) and \( u_t \), secrecy can generate additional volatility in the federal funds rate. In order to show that this is the case, the time path of \( n_t \) has to be derived endogenously from a description of the market learning process. This is done in the next section.

4. The Formation of Market Beliefs

The observation of the federal funds rate, together with the rest of the available information, enables market participants to gradually learn about the secret policy targets incorporated in \( N^T \). This learning process is described in this section by means of Bayes rules.
As a preliminary step, rewrite the closed form solution, equation (7), as:

\[ N^T \cdot e_t - u_t = z_t \]  
(8)

where

\[ z_t = \frac{1}{\pi_1} (F_t - \pi_2 B_{t-1} - \pi_4 n_t) \]  
(9)

and where the result that \( \pi_3 = -\pi_1 \) has been used. Since all the terms on the right hand side of (9) are known in period \( t \), \( z_t \) can be thought of as the information about \( N^T \) that becomes available to market participants in the course of period \( t \).

Suppose now that market beliefs about \( N^T \) at the beginning of period \( t \) (i.e., before the observation of \( z_t \) but after the observation of \( z_{t-1} \)) are adequately described by a normal prior distribution with mean \( n_{t-1} \) and precision \( h_{t-1} \). Then it follows from Bayes theorem that the posterior distribution of \( N^T \) after the observation of \( z_t \) is normal with mean:

\[ n_t = \frac{h_{t-1} n_{t-1} + rz_t}{h_{t-1} + r} \]  
(10)

and precision

\[ h_t = h_{t-1} + r \]  
(11)

where \( r = 1/(\sigma_u^2 + \sigma_e^2) \) (see for instance Cyert and De Groot (1974)).

Substituting (9) into (10) and solving for \( n_t \), we obtain the evolution of the market beliefs as a function of variables observed in the course of period \( t \) or earlier:

\[ n_t = \frac{\pi_1 h_{t-1} n_{t-1} + \tau (F_t - \pi_2 B_{t-1})}{\pi_1 h_{t-1} + (\pi_1 + \pi_4) r} \]  
(12)
Using (8), (10), and (11), it is possible to show that \( \lim_{t \to \infty} n_t = N^T \) (cf., Cyert and De Groot (1974)), so that, eventually, market beliefs will converge in probability to the true value of the policy target. However, since this target can be changed at every monthly FOMC meeting, the market learning process will start afresh after each one of these meetings. At any moment in time, therefore, market beliefs about the policy target will evolve according to equations (11) and (12). These beliefs will tend to sharpen over time, until a new FOMC meeting creates new uncertainty and forces financial markets to start their learning process again from a new set of priors.

5. The Consequences of Secrecy

This section combines all previous results to analyze the effects of secrecy on the equilibrium behavior of the federal funds rate and reserve aggregates.

In the absence of secrecy, the closed form solution (7) becomes:

\[
F_t = \pi_0 + (\pi_1 \pi_4) N^T + \pi_2 B_{t-1} + \pi_3 (e_t + u_t) \tag{13}
\]

With secrecy, using (8) and (10) into (7), the equilibrium behavior of the funds rate is given by:

\[
F_t = \pi_0 + \left[ \frac{\pi_1 (h_{t-1} + \tau)}{h_{t-1} + \tau} \right] N^T + \pi_2 B_{t-1} + \frac{\pi_4 h_{t-1}}{h_{t-1} + \tau} n_{t-1} \tag{14}
\]

\[
+ \left[ \frac{\pi_3 (h_{t-1} + \tau) - \pi_4 \tau}{h_{t-1} + \tau} \right] (e_t + u_t)
\]

Since \( \pi_4 < 0 \), a comparison of (13) and (14) yields immediately:

**Proposition 1:** Secrecy increases the contemporaneous reaction of the federal funds rate to shocks in the market for reserves.
An example can provide some intuition: Suppose that a positive shock to the demand for reserves occurs, increasing the equilibrium value of $F_t$. Market participants, who cannot observe the shock and ignore the true value of $N^T$, interpret the rise in $F_t$ as an indication that the true value of $N^T$ is lower than they previously thought (i.e., they reduce the posterior mean $n_t$). This in turn leads them to revise upward their expectation of the funds rate next period, $F_{t+1/t}$. As a result, borrowed reserves are reduced, and this puts further upward pressure on the current equilibrium value of $F_t$.

The contemporaneous reaction of the quantity of total reserves to the shocks is also affected by secrecy. Inserting (13) and (14) into (4) and then comparing the resulting expressions, one obtains:

Proposition 2: Secrecy increases the contemporaneous reaction of the quantity of total reserves to shocks originating on the supply side ($u_t$), and decreases its contemporaneous reaction to shocks originating on the demand side ($e_t$) of the market for reserves.

This asymmetry can be explained as follows. Secrecy increases the reaction of expectations to shocks originating on both sides of the market. These expectations enter the supply of reserves through the borrowing equation. Hence, secrecy amplifies the supply curve shift when a shock to either demand or supply occurs. This effect is destabilizing for both $F_t$ and the quantity of reserves in the face of a shock to supply ($u_t$). If however a shock to demand occurs ($e_t$), then the effect of secrecy is destabilizing on $F_t$ but stabilizing on the quantity of reserves (since the shock shifts the demand and supply curves in opposite directions). In other words, in the face of a demand shock secrecy induces $F_t$ to absorb a larger portion
of the shock, and as a result the equilibrium quantity of reserves changes by less.

A comparison of (13) and (14) also reveals that secrecy adds an extra random variable, $n_{t-1}$, to the closed form solution for $F_t$. Going through some tedious algebra, it is possible to show that, under an extremely plausible condition on the parameters of the borrowing equation (2), $\pi_2 > 0$ but $\pi_4 < 0$, the inclusion of $n_{t-1}$ among the fundamentals to which the funds rate reacts tends to add further variability to $F_t$. Thus, we have that:

**Proposition 3:** Secrecy tends to increase the unconditional variance of the federal funds rate.

The general intuition is the same as for Proposition 1. Observed values of $F_t$ convey information about both the temporary shocks and the permanent but unknown parameter $N^T$. Thus, secrecy induces market participants to interpret, say, an observed high value of $F_t$ as arising from a permanently lower than expected policy target $N^T$, even when it may be due to a temporary shock to the demand or supply for bank reserves. In other words, secrecy induces market participants to systematically underestimate the temporary nature of changes in $F_t$. Through the expectations mechanism (equation (5)), this amplifies the volatility of $F_t$.

The finding that the market overestimates the permanent nature of changes in $F_t$ descends directly from having modeled secrecy as generating parameter uncertainty. By contrast, some of the existing literature (Dotsey (1985), Rudin (1986)) has modeled secrecy as introducing exclusively some additional random noise in the policy reaction function. This extra random
noise has the effect of complicating the signal extraction problem solved by financial markets. In this case, the market underestimates (rather than overestimates) the permanent nature of changes in $F_t$, so that the reaction of $F_t$ to contemporaneous shocks is diminished. As a consequence, this literature concludes that secrecy can reduce the unconditional variance of the funds rate. This conclusion is not warranted. To the extent that some component of the secret policy targets is maintained roughly constant between one FOMC meeting and the next, it seems more appropriate to model secrecy as generating parameter uncertainty. In this case, secrecy unambiguously adds unconditional variability to the funds rate. Through the demand for total reserves (equation (4)), this additional volatility in $F_t$ then also tends to increase the unconditional variance of reserve aggregates.

Finally, and not surprisingly, secrecy also increases the variability of the conditional forecast error of the funds rate, $F_t - F_{t/t-1}$. Forming $F_{t/t-1}$ and subtracting it from the right hand side of (13), we have that in the absence of secrecy the conditional forecast error is:

$$F_t - F_{t/t-1} = \pi_3(e_t + u_t)$$  \hspace{1cm} (15)

By contrast, with secrecy:

$$F_t - F_{t/t-1} = \left[ \frac{\pi_1(h_{t-1} + r) + \pi_4}{h_{t-1} + r} \right] (N^T - n_{t-1})$$

$$+ \left[ \frac{\pi_3(h_{t-1} + r) - \pi_4}{h_{t-1} + r} \right] (e_t + u_t)$$  \hspace{1cm} (16)

Comparing the right hand side of (15) and (16), it follows immediately that secrecy increases the variance of the conditional forecast error. This finding conforms to intuition: improving the information set of market participants enables them to make more accurate forecasts.
As already mentioned in Sections 2 and 3, these results are robust to several possible extensions of the underlying model. In particular, it is possible to add serially correlated shocks to the demand for total reserves (equation (4)), or to the borrowing function (equation (2)), or to change the hypothesis concerning the distribution of the market prior beliefs, without affecting the contents of Propositions 1-3 in any substantial respect. To summarize, there are really only two crucial ingredients in the model. One is that in equilibrium $F_t$ depends on $F_{t+1}/t$ — cf., equation (5). The other is that secrecy generates uncertainty about a (temporarily) stable feature of the policymaker behavior — i.e., parameter uncertainty. The resulting learning process about the undisclosed policy targets adds volatility to the market expectations, and thus ultimately to the federal funds rate itself.

6. Conclusions

The Federal Reserve has recently been criticized for refusing to disclose its short run monetary policy decisions. One of the arguments used by the Federal Reserve in defence of secrecy has been that public disclosure of the short run policy targets would add variability to interest rates. This paper has investigated the validity of the Federal Reserve argument by analyzing a simple theoretical model of the market for bank reserves. Secrecy has been modelled by assuming that financial markets are uncertain about a parameter in the Federal Reserve reaction function.

The model contradicts the argument raised by the Federal Reserve against public disclosure: secrecy tends to increase the unconditional and the conditional variance of the federal funds rate, as well as the contemporaneous reaction of the funds rate to shocks to the demand or supply of bank
reserves. This additional volatility of the funds rate may increase the variability of reserve aggregates. Moreover, through a term structure relationship, this additional volatility can extend to interest rates of longer maturity and to other financial aggregates. As a result, secrecy could make it more difficult for the Federal Reserve to achieve its monetary objectives.

Naturally, it is difficult to derive precise normative implications from these results since the model is not derived from the behavior of maximizing individuals. Moreover, there are other arguments in favor or against secrecy in addition to those concerning interest rates and reserve variability - both for short run and long run monetary policy decisions (see Goodfriend (1986), Cukierman and Meltzer (1986), King (1984)). A careful analysis of these other aspects of the secrecy issue is left for future research.
FOOTNOTES

*This paper was written while I was Assistant Professor at Stanford University. A previous version circulated under the title "Public Disclosure of the Money Targets: Does it Hinder Monetary Control?". I wish to thank two anonymous referees, Giovanna Mossetti, Mike Dotsey and participants in a workshop at Stanford University for helpful comments.

1 Somewhat more complicated versions of the same basic model can be found, for instance, in Goodfriend et al. (1986) and in Dotsey (1985). As discussed in the text, the results of this paper can be extended to those versions.

2 In principle, the equation should also include $\sum_{i=2}^{\infty} F_{t+i/t}$ as explanatory variables -- see Goodfriend (1983). Here they have been omitted in order to simplify the computations.

3 Equation (5) has been obtained by plugging (2) and (3) into (1) and then equating the resulting expression with (4).

4 The condition is that $\delta_1 > |\delta_2|$. In other words, the supply of borrowed reserves (equation (2)) must be relatively more sensitive to $F_t$ than to $F_{t+1/t}$.

5 Unlike in the case of the unconditional variance, this result is obtained also by Dotsey (1985) and Rudin (1986).
APPENDIX

Computation of the coefficients of equation (7).

Advance (7) by one period and take expectations, using (2) and (7) to simplify:

\[
F_{t+1/t} = \frac{1}{1-\pi_2 \delta_2} \left[ (\pi_0 + \delta_0 \pi_2 + \delta_1 \pi_0 \pi_2) + (\pi_1 + \pi_4 + \delta_1 \pi_2 \pi_4) \pi_t + \pi_3 + \delta_1 \pi_2 B_{t-1} + \delta_1 \pi_2 \pi_3^N + \delta_1 \pi_2 \pi_3 (u_t + e_t) \right]
\] (A.1)

Substitute (A.1) into (5) and equate coefficients with (7):

\[
\pi_0 = \frac{1}{\gamma + \delta_1 - \alpha_1} \left[ \alpha_1_0 \delta_0 \delta_2 \frac{(\pi_0 + \delta_0 \pi_2 + \delta_1 \pi_0 \pi_2)}{1-\pi_2 \delta_2} \right]
\]

\[
\pi_1 = \frac{1}{\gamma + \delta_1 - \alpha_1} \left[ -1 - \frac{\delta_2 \delta_1 \pi_2}{1-\pi_2 \delta_2} \right]
\]

\[
\pi_2 = \frac{1}{\gamma + \delta_1 - \alpha_1} \left[ -\delta_3 - \frac{\delta_2 (\delta_3 + \delta_1 \pi_2)}{1-\delta_2 \pi_2} \right]
\]

\[
\pi_3 = \frac{1}{\gamma + \delta_1 - \alpha_1} \left[ 1 - \frac{\delta_2 \delta_1 \pi_2 \pi_3}{1-\pi_2 \delta_2} \right]
\]

\[
\pi_4 = \frac{1}{\gamma + \delta_1 - \alpha_1} \left[ -\delta_2 \frac{(\pi_1 + \pi_4 + \delta_1 \pi_2 \pi_4)}{1-\pi_2 \delta_2} \right]
\]

Solve for \( \pi_2 \) first, to obtain:

\[
\pi_2 = \frac{(\alpha_1 - \gamma - \delta_1) + \sqrt{\Delta}}{2\delta_2 (\alpha_1 - \gamma)} > 0
\]

where:

\[
\Delta = (\gamma + \delta_1 - \alpha_1)^2 + 4\delta_2 \delta_3 (\gamma - \alpha_1) > 0
\]
and where the positive root has been chosen in conformity with McCallum (1983) criterion of choosing the minimal set of state variables: when \( \delta_3 = 0 \), I want \( \pi_2 = 0 \), which in turn forces me to choose the positive root.

Solving recursively for the remaining coefficients, we obtain:

\[
\pi_0 = \frac{\alpha_0(\alpha_1+\delta_1-\gamma-\sqrt{\Delta}) + 2\delta_0(\gamma-\alpha_1)}{(\alpha_1-\gamma)(\gamma+\delta_1-\alpha_1+\sqrt{\Delta}+2\delta_2)}
\]

\[
\pi_1 = -\pi_3 = \frac{\gamma-\alpha_1-\delta_1+\sqrt{\Delta}}{(\alpha_1-\gamma)(\gamma+\delta_1-\alpha_1+\sqrt{\Delta})} < 0
\]

\[
\pi_4 = \frac{2\delta_2(\gamma-\alpha_1-\delta_1+\sqrt{\Delta})}{(\gamma-\alpha_1)(\gamma+\delta_1-\alpha_1+\sqrt{\Delta})(\gamma+\delta_1-\alpha_1+\sqrt{\Delta}+2\delta_2)} < 0
\]
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