TARIFFS, TERMS OF TRADE, AND THE REAL EXCHANGE RATE IN AN INTERTEMPORAL OPTIMIZING MODEL OF THE CURRENT ACCOUNT

By

Sebastian Edwards
University of California, Los Angeles

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Abstract

In this paper a minimal general equilibrium intertemporal model, with optimizing consumers and producers, is developed to analyze the process of real exchange rate determination. The model is completely real, and considers a small open economy that produces and consumes three goods each period. The model is also used to analyze the way in which the current account responds to several shocks. The working of the model is illustrated for the case of two disturbances: the imposition of import tariffs, and external terms of trade shocks. In the case of import tariffs, a distinction is made between temporary, anticipated, and permanent changes. It is shown that, without imposing rigidities or adjustment costs, interesting paths for the equilibrium real exchange rate can be generated. In particular "overshooting" and movements in opposite directions in periods one and two can be observed. Precise conditions under which temporary import tariffs will improve the current account are derived. Finally, several ways in which the model can be extended to take into account other issues such as changes in the fiscal deficit, and financial deregulation are discussed in detail.

Sebastian Edwards
Dept of Economics
UCLA
Los Angeles, CA 90024
(213)-825-5304
I. Introduction

The recent wide fluctuations experienced by real exchange rates (RERs) in the U.S. and other countries have generated concern among policymakers and academics. In fact, in the last two years or so policy analyses have increasingly focused on issues related to real exchange rate disequilibrium, and some proposals aimed at actively intervening in the exchange market in order to reduce real exchange rate "misalignment" have been discussed.\(^1\)

These proposals, however, would only make sense if recent RER movements in fact represent a disequilibrium phenomenon, where the actual RER exhibits sustained departures from its equilibrium value. If, on the other hand, this is not the case, and observed RER changes respond to "fundamentals," intervention could have counterproductive effects. It would seem, then, that in order to fully understand RER behavior, and to propose policy actions, it is first necessary to have a fully articulated theory on how the equilibrium value of this relative price responds to different (real) disturbances. Most of the recent exchange rates research, however, has focused on nominal exchange rate determination, tending to ignore real aspects of real exchange rate behavior.\(^2\)

The purpose of this paper is to propose a minimal real model suitable for analyzing how equilibrium RERs respond to different (real) shocks. The functioning of the model is illustrated for the case of two disturbances: changes in import tariffs and terms of trade shocks. However, the way in which the minimal model can be expanded to analyze other disturbances is illustrated in detail at the end of the paper.\(^3\)

The model presented in this paper considers the case of a small open economy where optimizing producers and consumers produce and consume three goods -- importables, exportables and nontradables. Foreign borrowing is
allowed, and the only constraint faced by the nationals of this country is that the present value of the current account balances equals zero. There is no uncertainty, and agents have perfect foresight. The model is completely real and is solved using duality theory.

The paper is organized as follows. In Section II a very general intertemporal general equilibrium model of a (small) real economy with optimizing consumers and producers is developed. Here the concept of equilibrium RER in an intertemporal setting is discussed, and the modeling strategy is set forward. Section III deals with changes in import tariffs (temporary, permanent and anticipated) and their effect on equilibrium RERs. Here, the effect of temporary tariffs on the current account is also analyzed. It is shown that whether this type of policy will result in a worsening or in an improvement of the current account will depend on the different intertemporal elasticities. In Section IV the impact of changes in the international terms of trade on the path of equilibrium RERs is analyzed. The results obtained are then compared to those of the tariff case. In Section V the effect of terms of trade changes on the current account are investigated. The analysis presented here is essentially an extension of the Laursen-Metzler model to the intertemporal case with nontradables. Section VI deals with extensions. It is shown that the model is general enough as to handle a large number of issues, including the welfare effects of alternative policy packages dealing with economic deregulation. Section VII contains the concluding remarks.

II. The Model

Although the framework used is general enough as to accommodate many goods and factors, it is useful to think of this economy as producing three
goods -- exportables (X), importables (M) and nontradables (N) -- using standard technology, under perfect competition. It is assumed that there are more factors than tradable goods, so that factor price equalization does not hold. One way to think of this is by assuming that capital is sector specific, while labor can move freely across all three sectors. Alternatively, one can think that this economy uses capital, labor and two different types of land.

In this version of the model there is no investment, capital accumulation or growth (see, however, Section V). We consider two periods only -- periods 1 and 2. Residents of this country can borrow or lend internationally at the given world rate of interest. There are no exchange controls, and the only constraint is that at the end of period 2 the country has paid its debts. The importation of M is subject to specific import tariffs both in periods 1 and 2. Since there is no investment, the current account is exactly equal to savings in each period. If the residents of this country dis-save in period 1, their expenditure will exceed their income, and the corresponding current account deficit will be financed through borrowing from abroad. On the preferences side, it is assumed that the utility function is time weakly separable, with preferences in each period being homothetical. This means that consumer's optimization takes place in two stages. Given prices and the discount factor, consumers first decide how to allocate expenditure across periods. In the second stage, they decide how to allocate each period's expenditure across the three goods. These assumptions regarding preferences turn out to be very convenient, since they permit the use of within-period price indexes, as in Svensson and Razin (1983) and Edwards and van Wijnbergen (1986). The nominal exchange rate is fixed and equal to one. The price of X is taken
as the numeraire.

The model is worked out using duality theory and is given by equations (1) through (5). Superscripts refer to periods (i.e., $R^2$ is the revenue function in period 2); subscripts refer to partial derivatives with respect to that variable (i.e., $R^1_{q^1}$ is the partial derivative of period 1's revenue function relative to $q^1$ (the price of nontradables in period 1); $R^2_{q^2 p^2}$ is the second derivative of $R^2$ with respect to $q^2$ and $p^2$):

$$R^1(1, p^1, q^1, v) + \delta^* R^2(1, p^2, q^2, v) + r^1(E_{p^1} - R^1_{p^1}) + \delta^* r^2(E_{p^2} - R^2_{p^2}) - E[\pi^1(1, p^1, q^1), \delta^* \pi^2(1, p^2, q^2), w]$$

(1)

$$R^1_{q^1} = E$$

(2)

$$R^2_{q^2} = E$$

(3)

$$p^1 = p^{1*} + r^1$$

(4)

$$p^2 = p^{2*} + r^2$$

(5)

where the following notation is used:

$R^i(\_); i = 1, 2$ Revenue functions in period $i$. Their partial derivatives with respect to each price are equal to the supply functions.

$p^i; i = 1, 2$ Domestic relative price of imports in period $i$.

$q^i; i = 1, 2$ Relative price of nontradables in period $i$.

$V$ Vector of factors of production, assumed to be fixed.

$r^i; i = 1, 2$ Specific tariffs in period $i$.

$\delta^*$ World discount factor, equal to $(1+r^*)^{-1}$, where $r^*$ is world real interest rates (in terms of tradables).
E( ) \text{Intertemporal expenditure function.} \\
\pi^i(p^i, q^i) \text{Exact price indexes, which under assumptions of homothecity and separability, corresponds to unit expenditure functions.} \\
\text{(See Edwards and van Wijnbergen, 1986.)} \\
W \text{Total aggregate welfare.} \\

Equation (1) is the intertemporal budget constraint, and states that present value of income -- generated through revenues from production \( R^1 + \delta * R^2 \), plus tariffs collection -- had to equal present value of expenditure. Given the assumption of perfect access to the world capital market, the discount factor used in (1) is the world discount factor \( \delta * \). Equations (2) and (3) are the equilibrium conditions for the nontradables market in periods 1 and 2; in each of these periods the quantity supplied of \( N (\frac{R^1}{q^1} \text{ and } \frac{R^2}{q^2}) \) has to equal the quantity demanded. Given the assumptions about preferences (separability and homothecity) the demand for \( N \) in period \( i \) can be written as:

\[
E_i = E_i \pi^i q^i \\
\text{(6)}
\]

Equations (4) and (5) specify the relation between domestic prices of imports, world prices of imports and tariffs.

The current account in period 1 is equal to the difference between income and total expenditure in that period:

\[
CA^1 = R^1(\pi) + \pi^1 (\frac{R^1}{p^1} - E^1) - E^1 \pi^1 \\
\text{(7)}
\]

II.1 \textbf{The Concept of Equilibrium Real Exchange Rates}

In models with importables and exportables the definition of "the" real exchange rate becomes "tricky", since the by-now traditional concept of
relative price of tradables to nontradables loses some meaning. The reason, of course, is that if there are shocks that affect the price of X relative to M, it is not possible to talk about the Hicksian composite "tradables" anymore. In a way, in this type of model there are two RERs: the relative price of exportables to nontradables, and the relative price of importables to nontradables \( \frac{1}{q} \). For this reason, and in order to simplify the exposition, in this paper we will focus on the (inverse) of real exchange rate for exports \( q \). Of course, once it is known how \( q \) responds to changes in fundamentals, it is possible to compute the effect of shocks on any of the traditional indexes of RER change.

In the intertemporal model presented above there is not one equilibrium value of the real exchange rate, but rather a path of equilibrium RERs. Within this intertemporal framework the equilibrium RER in a particular period is defined as the inverse of \( q \) that, for given values of other variables such as world prices, technology and tariffs, equilibrates simultaneously the external and internal (i.e., nontradables) sectors. In terms of the model the vector of equilibrium relative prices \( \bar{q} = (q_1, q_2) \) is composed of those \( q^i \)'s that satisfy equations (1) through (5), for given values of the other fundamental variables. In that regard, since the system given by equations (1)-(5) depicts a full equilibrium -- both intertemporal for the external sector -- and period-by-period for the nontradables market, the initial \( q \)'s are the (inverse of the) equilibrium real exchange rates for periods 1 and 2.

From the inspection of equations (1)-(5) it is apparent that exogenous shocks in, say, the international terms of trade, will affect the vector of equilibrium RERs through two interrelated channels. The first one, which has been subject to some discussion in the literature, is related to intratempo-
ral effects of terms of trade shocks on resource allocation and consumption decisions. For example, as a result of a temporary worsening of the terms of trade, there will be a tendency to produce more and consume less of $M$ in that period. This, plus the income effect resulting from the worsening of the terms of trade will generate an incipient disequilibrium in the nontradables market which will have to be resolved by a change in the equilibrium $q$. In fact, if we assume that there is an absence of foreign borrowing these intratemporal effects will be the only relevant ones. However, with capital mobility, as in the current model, there is a second intertemporal channel through which changes in exogenous variables will affect the vector of equilibrium RERs. For example, in the case of a temporary worsening of the terms of trade, the consumption discount factor $\pi^2 \delta^*/\pi^1$ will be affected, altering the intertemporal allocation of consumption. In the rest of the paper we will emphasize the role of this intertemporal effect.

II.2. The Solution of the Model

Equations (1)-(5) can be manipulated to find out how the vector of equilibrium RERs responds to exogenous shocks such as changes in tariffs, disturbances to the international terms of trade, international transfers, and changes in world interest rates. From (1)-(5) and (7) the reaction of the current account to these types of shocks can also be found. Differentiating (1)-(5) we can write:

$$
\begin{bmatrix}
\epsilon_1 & \epsilon_2 & -\epsilon_3 \\
\begin{bmatrix}
R_{11}^1 q_{11}^1 & E_{11}^1 q_{11}^1 \\
q_{11}^1 q_{11}^1 & q_{11}^1 q_{11}^1 \\
\end{bmatrix} & -E_{1}^2 q_{11}^1 & -\pi_1^1 E_{1}^1 q_{11}^1 \\
-E_{1}^2 q_{11}^2 & \begin{bmatrix}
R_{22}^2 q_{2}^2 & E_{2}^2 q_{2}^2 \\
q_{2}^2 q_{2}^2 & q_{2}^2 q_{2}^2 \\
\end{bmatrix} & -\pi_2^2 E_{2}^2 q_{2}^2 \\
q_{11}^2 & q_{2}^2 & q_{2}^2 q_{2}^2 \\
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
dq_1 \\
dq_2 \\
dW \\
\end{bmatrix}
= \begin{bmatrix}
\beta_1 & \beta_2 & \beta_3 & \beta_4 \\
\gamma_1 & 0 & 0 & \gamma_2 \\
\alpha_1 & 0 & 0 & \alpha_2 \\
\end{bmatrix}
\begin{bmatrix}
dp_1 \\
dr_1 \\
dr_2 \\
\end{bmatrix}
$$
where, as already noted, subindexes stand for partial derivatives with respect to that particular variable (i.e., \( R_{1 \rightarrow 1}^{\frac{p}{p}} \) is the slope of the supply curve for \( M \) in period 1.)

\[
E_{i'j'} \quad E_{i'j'} \quad E_{i'j'}
\]

\[
q''q' \quad p''p' \quad p''p'
\]

for \( i \neq j \), are intra and intertemporal cross demand effects whose exact expressions are given in Appendix 1. \( R_{1 \rightarrow 1}^{\frac{p}{p}} \), \( R_{2 \rightarrow 2}^{\frac{p}{p}} \), and \( R_{2 \rightarrow 2}^{\frac{q}{q}} \) are the slopes of the supply curves for \( M \) and \( N \) in periods 1 and 2, and are thus positive.

\[
E_{1 \rightarrow 1}^{\frac{q}{q}} \quad E_{1 \rightarrow 1}^{\frac{q}{q}} \quad E_{2 \rightarrow 2}^{\frac{p}{p}} \quad E_{2 \rightarrow 2}^{\frac{p}{p}}
\]

\[
q''q' \quad q''q' \quad p''p' \quad p''p'
\]

are the slopes of the compensated demand curves for \( N \) and \( M \) in periods 1 and 2 and are negative. (For the exact expressions see Appendix 1.) \( E_{1 \rightarrow 1}^{\frac{p}{p}} \) and \( E_{2 \rightarrow 2}^{\frac{p}{p}} \) capture the income effects in periods 1 and 2; \( \pi_{1}^{\frac{q}{q}} \) and \( \pi_{2}^{\frac{q}{q}} \) are unitary demands (or expenditure shares) for \( N \) in periods 1 and 2.

Also, the following notation has been used:

\[
\epsilon_{1} = \tau^{1} \left[ E_{1 \rightarrow 1}^{\frac{1}{1}} - R_{1 \rightarrow 1}^{\frac{1}{1}} \right] + \delta \tau^{2} \left[ E_{2 \rightarrow 2}^{\frac{2}{2}} - R_{2 \rightarrow 2}^{\frac{2}{2}} \right] + \tau^{1} \left[ E_{1 \rightarrow 2}^{\frac{1}{2}} + \delta \tau^{2} \left[ E_{2 \rightarrow 2}^{\frac{2}{2}} - R_{2 \rightarrow 2}^{\frac{2}{2}} \right] + \tau^{1} \left[ E_{1 \rightarrow 2}^{\frac{2}{2}} \right]
\]

\[
\epsilon_{2} = \delta \tau^{2} \left[ E_{2 \rightarrow 2}^{\frac{2}{2}} - R_{2 \rightarrow 2}^{\frac{2}{2}} \right] + \tau^{1} \left[ E_{1 \rightarrow 2}^{\frac{2}{2}} \right]
\]

\[
\epsilon_{3} = E_{w} \left[ 1 - \tau^{1} \left[ E_{1 \rightarrow 2}^{\frac{1}{1}} \right] \right] + \delta \tau^{2} \left[ E_{2 \rightarrow 2}^{\frac{2}{2}} - R_{2 \rightarrow 2}^{\frac{2}{2}} \right]
\]

\[
\beta_{1} = \left[ \tau^{1} \left( R_{p \rightarrow 1}^{\frac{p}{p}} - E_{p \rightarrow 1}^{\frac{p}{p}} \right) + \delta \tau^{2} \left[ E_{2 \rightarrow 2}^{\frac{p}{p}} - R_{2 \rightarrow 2}^{\frac{p}{p}} \right] \right] ; \quad \beta_{2} = -\left( E_{1 \rightarrow 1}^{\frac{p}{p}} \right)
\]

\[
\beta_{3} = -\delta \tau^{2} \left[ E_{p \rightarrow 2}^{\frac{p}{p}} - R_{p \rightarrow 2}^{\frac{p}{p}} \right] ; \quad \beta_{4} = \left[ \delta \tau^{2} \left[ E_{2 \rightarrow 2}^{\frac{p}{p}} - R_{2 \rightarrow 2}^{\frac{p}{p}} \right] + \tau^{1} \left( R_{p \rightarrow 1}^{\frac{p}{p}} - E_{p \rightarrow 1}^{\frac{p}{p}} \right) \right] \quad \epsilon_{1} \frac{1}{p \rightarrow 1} \frac{1}{p \rightarrow 1} \frac{1}{p \rightarrow 1}
\]

\[
\gamma_{1} = \left( E_{1 \rightarrow 1}^{\frac{1}{1}} R_{1 \rightarrow 1}^{\frac{1}{1}} \right) ; \quad \gamma_{2} = \left( E_{1 \rightarrow 1}^{\frac{1}{1}} R_{1 \rightarrow 1}^{\frac{1}{1}} \right)
\]

\[
\alpha_{1} = \left( E_{p \rightarrow 1}^{\frac{1}{1}} R_{p \rightarrow 1}^{\frac{p}{p}} \right) ; \quad \alpha_{2} = \left( E_{p \rightarrow 1}^{\frac{1}{1}} R_{p \rightarrow 1}^{\frac{p}{p}} \right)
\]

One of the consequences of this intertemporal model is that since there are actually six goods -- \( X, M \) and \( N \) in periods 1 and 2 -- there is room for a large combination of substitution effects on demand -- both intra- and intertemporal -- that make the signing of some of the terms in (8)
impossible without making further assumptions. Intertemporal substitution takes place via \( E_{11} \), \( E_{12} \), and \( E_{22} \). For example, \( E_{12} \) is the response of (real) consumption on all goods in period 1 to changes in period 2's (exact) price index. Notice, however, that given the two periods nature of this model, and the assumption of time separability of the utility function there is gross substitutability of (all) goods across both periods, so that \( E_{12} \) is unambiguously positive.

Since the price indexes \( \pi_1 \) and \( \pi_2 \) are unit expenditure functions, their derivatives with respect to the different prices are positive and are interpreted as consumption shares. Intratemporal cross demand effects, however, can be either positive or negative. The source of this indeterminacy stems both from the possibility of within period gross substitutability or complementarity for any pair of goods, and from intertemporal substitution in consumption. Take for example, the case of the cross price effect between importables and nontradables in period 1, \( E_{11} = (E_{11} \pi_{11} + \pi_1 E_{11} \pi_{11}) \). If \( N \) and \( M \) are gross substitutes in period 1, \( \pi_{11} > 0 \). However, since \( E_{11} < 0 \) by concavity of the expenditure function, the second term in the RHS can dominate, and \( E_{11} < 0 \). The reason for this is that an increase in the price of \( M \) in period 1 will have two opposite effects on the demand for \( N \) in that period. First, a higher \( p^1 \) will encourage consumption of nontradables in period 1 via the intratemporal effect \( \pi_{11} \). Second, the higher \( p^1 \) will generate an intertemporal reallocation away from consumption (on all goods) in period 1. Depending on which of these two effects dominate \( E_{11} \geq 0 \).

Given our assumption of intertemporal separable preferences, all intertemporal cross effects will be positive (i.e., \( E_{12} > 0 \), \( E_{12} > 0 \)). Also, from the properties of revenue and expenditure functions we know that
III. Tariffs, Equilibrium Real Exchange Rates and the Current Account

The traditional international and development literatures have analyzed the effects of tariff changes on the equilibrium real exchange rate from two perspectives. First, in the shadow pricing literature it has been argued that the shadow price of foreign exchange can be approximated by the (real) exchange rate under free trade. Along these lines authors have investigated how the RER will be affected if all trade restrictions are lifted (Taylor 1978). Second, the policy literature on international trade liberalization and reform has discussed the way in which the lowering of tariffs and relaxation of other trade impediments will affect the equilibrium real exchange rate (Balassa 1982). The standard result from most of the work along both of these traditions is that the imposition of an import tariff will tend to improve the current account and will require an appreciation of the equilibrium real exchange rate. However, a shortcoming of this literature is that it has generally used partial equilibrium models without nontradables, and has ignored intertemporal considerations. As a result, these models have not been able to tackle important questions such as the effects of temporary or unanticipated shocks on the real exchange rate and current account.

III.1. Temporary Changes in Tariffs

In this section we investigate the effects of a temporary change in period 1's tariff on the vector of equilibrium real exchange rates and on the capital account in period 1. In order to simplify the notation assume that initially tariffs in period 1 and 2 are equal to \( r^1 = r^2 = r \).
We first discuss the case where initial tariffs are equal to zero, and then more to the more general case of positive initial tariffs. From (8), setting \( dr^1 = dp^1 \), \( dr^2 = dp^2 = 0 \) and evaluating for initial \( \tau_s = 0 \), we obtain the following expressions for changes in the equilibrium relative prices in periods 1 and 2:

\[
\frac{dq_1^1}{dr^1} = -\frac{E}{\Delta^1} \left\{ \left( \frac{E}{R_{pq}^1} - \frac{E}{R_{pq}^2} \right) \left( \frac{E}{q_{pq}^1} - \frac{E}{q_{pq}^2} \right) + \frac{E}{q_{pq}^1} \frac{E}{q_{pq}^2} \right\}
\]

(9)

where

\[
\Delta^1 = -E \left[ \left( \frac{R_{pq}^1}{q_{pq}^1} - \frac{E}{q_{pq}^1} \right) \left( \frac{R_{pq}^2}{q_{pq}^2} - \frac{E}{q_{pq}^2} \right) - \frac{E}{q_{pq}^1} \frac{E}{q_{pq}^2} \right] < 0.
\]

Notice that (9) involves substitution effects only. The reason is that by assuming zero initial tariffs we do not have first order income effects. The sign of (9) is undetermined, since \( E_{p_{pq}^1} > 0 \). If, however, it is assumed that importables and nontradables are gross substitutes in period 1 \( (E_{p_{pq}^1} > 0) \) we obtain:

\[
\frac{dq_1^1}{dr^1} > 0.
\]

This assumption of gross substitutability in period 1 requires that \( E_{p_{pq}^1} \), \( E_{p_{pq}^2} > 0 \) and \( |E_{p_{pq}^1} E_{p_{pq}^2}^1| < E_{p_{pq}^1} E_{p_{pq}^2}^1 \). Only in this case, then, we have that the more traditional result that suggests that higher tariffs induce an equilibrium real depreciation, will hold. The intuition for this result is rather simple. With no income effects and gross substitutability in demand everywhere, the increase in period 1 import prices generated by the imposition of the tariff, will result in a reduction in the demand for \( M \) in that period, and an increase in demand for all other goods including nontradables in that period. As a consequence both production and the
relative price of $N$ in period 1 will increase.

Assuming no first order income effect, the change in period 2 equilibrium price of nontradables will be affected by the temporary period 1 tariff in the following way:

\[
\frac{dq_2^2}{dr_1^1} = - \frac{E_w}{\Delta_1} \left\{ E_{2q_1} \left\{ E_{1p_1} \left( p_{q_1} \cdot R_{p_1}^1 \right) - p_{q_1} \right\} + E_{1q_1} \left\{ p_{q_1} \cdot R_{q_1}^1 \right\} \right\},
\]  

(10)

Notice that in this intertemporal model a temporary tariff imposition in period 1 only will affect the equilibrium RER in future periods. This, of course, is only possible in a model with borrowing, where agents can use the international capital market to smooth the effects of foreign shocks through time. If $E_{2q_1}^1 = 0$ in (10), then there is no intertemporal substitution and $(dq^2_2/dr_1^1) = 0$. If however, $E_{2q_1}^1 > 0$, and under the assumption of gross substitutability everywhere $(dq^2_2/dr_1^1)$ is positive, indicating that a temporary tariff in period 1 will result in an equilibrium appreciation in period 2. The intuition in this case is analogous to the case discussed above.

Let us consider now the case when tariffs are initially greater than zero. Now, we will have first order income effect and a temporary tariff will affect $q_1^1$ and $q_2^1$ in the following way:
\[
\frac{dq_1}{dr_1} = \frac{1}{\Delta} \left\{ r \left[ (R_{1p}^{1p} - E_{1p}^{1p} - \delta^* E_{1p}^{1p} p_{2p}^{2p} ) \left\{ E_{1q}^{1q} \pi_{1q}^{1q} - \pi_{1q}^{1q} \pi_{1q}^{1q} \right\} \right] + \pi_{1q}^{1q} E_{1q}^{1q} \left[ R_{2q}^{2q} - E_{2q}^{2q} \right] \right\} - \frac{1}{q_{1q}^{1q}} \pi_{1q}^{1q} \left( E_{1q}^{1q} \pi_{1q}^{1q} E_{1q}^{1q} \pi_{1q}^{1q} \right) \right\} \geq 0, \quad (11)
\]

and
\[
\frac{dq_2}{dr_1} = \frac{1}{\Delta} \left\{ r \left[ (R_{1p}^{1p} - E_{1p}^{1p} - \delta^* E_{1p}^{1p} p_{2p}^{2p} ) \left\{ E_{1q}^{1q} \pi_{1q}^{1q} - \pi_{1q}^{1q} \pi_{1q}^{1q} \right\} \right] + \pi_{1q}^{1q} E_{1q}^{1q} \left[ R_{2q}^{2q} - E_{2q}^{2q} \right] \right\} - \frac{1}{q_{1q}^{1q}} \pi_{1q}^{1q} \left( E_{1q}^{1q} \pi_{1q}^{1q} E_{1q}^{1q} \pi_{1q}^{1q} \right) \right\} \geq 0 \quad (12)
\]

where \( \Delta \) is the determinant of the LHS matrix in (8), which under usual stability requirements is negative (see Appendix 2.)

Equations (11) and (12) provide a number of important results. First, they show that in this general case, contrary to the more traditional shadow pricing and trade reform literature temporal changes in tariffs don't necessarily result in an equilibrium real depreciation. In fact the signs of \( dq_1/dr_1 \) and \( dq_2/dr_1 \) are undetermined. Second, in the current model there are income effects which can, and generally will, operate in the opposite direction than the substitution effect. The importance of the income effects will depend on the initial levels of the tariffs \( r_1 \) and \( r_2 \) and on \( E_w, E_{2w}, \) and \( E_{1w}. \)
Another important aspect of (11) and (12) is that the equilibrium path of the real exchange rate (RER) can be characterized by "overshooting," where \( q^1 \) increases by more than \( q^2 \). Moreover, it is possible that as a result of the temporary tariff increase \( q^1 \) and \( q^2 \) will move in opposite directions. This, of course, makes the evaluation of actual movements of RER's, and the determination of whether they represent equilibrium or disequilibrium movement, particularly difficult. Notice that this type of behavior of the RERs is not the result of price rigidities, but rather responds to different values of the elasticities in different periods.

In sum, then, in this general equilibrium intertemporal setting with foreign borrowing it is not possible to determine a priori whether temporary tariff hikes will appreciate or depreciate the equilibrium real exchange rate. Moreover, in this model it is possible to obtain several "puzzling" time path of the RERs, including overshooting, or even movements in opposite directions. This result is in contradiction to the more traditional, and generally accepted, policy oriented literature on tariff reforms and shadow pricing.

III.2 Temporary Tariffs and the Current Account

An important question relates to the way in which the current account will react to the imposition of a temporary tariff. From equation (7) we can derive the following expression for the case of very low (or zero) initial tariffs:

\[
\frac{dCA^1}{dr^1} = -\pi^1 \frac{E}{\pi^1} \frac{\pi^1 p^1}{\pi^1 q^1} (\frac{dq^1}{dr^1}) - \delta \pi^2 \frac{E}{\pi^2} \frac{\pi^2 q^2}{\pi^2 q^2} (\frac{dq^2}{dr^2}).
\]

Equation (13) provides a general expression for the response of the current account in period 1 to the imposition of a temporary tariff that
affects period 1 only. Notice that the presence of an \( E_{\pi_1} \) in every one of the RHS terms of equation (13) clearly highlights the fact that tariff changes will only affect the current account via intertemporal channels.  

The first term in the RHS of equation (13) is positive and captures the direct effect of the imposition of a tariff in period 1 on the current account in that period. The intuition is straightforward. The higher period one tariff makes period 1 consumption relatively more expensive, and as a result of this the public substitutes consumptions away from period 1 into period 2, generating an improvement of the current account balance in period 1. The magnitude of this effect will depend both on the intertemporal direct effect \( E_{\pi_1} \) and on the initial share of imports on period 1 expenditure \( \pi_1^P \).  

The second and third terms on the RHS of equation (13) are indirect effects, that via operate changes in periods 1 and 2 RERs. Since, as was established above, the signs of \( (dq_1^1/dr_1^1) \) and \( (dq_2^2/dr_1^1) \) cannot be determined a priori, the sign of these two terms in (14) are generally undetermined. However, their interpretation is quite straightforward within the intertemporal framework of the current model. If the temporary tariff results in an equilibrium appreciation in period 1, \( (dq_1^1/dr_1^1) > 0 \), there will be an additional force towards a current account improvement. The reasoning is again simple. If the tariff results in a higher equilibrium price of nontradables in period 1, there will be substitution away from period 1 expenditure, generating a further improvement in the current account in that period. The third term on the RHS relates the change in period's 2 RER to periods 1 current account. If as a consequence of the tariff \( q_2^2 \) increases (see equation (10) above for the conditions under which this will take place), there will be a tendency to substitute expenditure
away from period 2 into period 1, generating forces that will tend to worsen period 1's current account. In sum, then, the effects of imposing a (temporary) import tariff on period 1's current account is undetermined, and will depend on the strength of the intertemporal price effects, initial expenditure on importables and nontradables, and on the effects of the tariff on the RER vector. This results contrasts sharply with the traditional static view where the conditions for tariffs improving the current account are related to imports and exports demand elasticities within each period.\(^{10}\)

If we assume that tariffs are initially positive, the equation for period 1 current account becomes:

\[
\frac{dCA}{dr^1} = \left\{ r^1 \left( E_{11}^{1,1} - R_{11}^{1,1} \right) - \pi^1 E_{11}^{1,1} \pi^1 \right\} \\
+ \left\{ r^1 \left( E_{11}^{1,1} - R_{11}^{1,1} \right) - \pi^1 E_{11}^{1,1} \pi^1 \right\} \frac{dq_{11}}{dr^1} \\
+ \left[ r E_{12}^{1,2} - \delta^* \pi_{22}^1 E_{12}^{1,2} \right] \frac{dq_{22}}{dr^1} \\
+ E_{1\pi}^W \left\{ r \pi_{11}^1 - \pi_{11}^1 \right\} \frac{dW}{dr^1}
\]

(14)

where \(dW/dr^1\) is the welfare effect of the temporary hike in the tariff, and is negative. Not too surprisingly, given our previous discussions, equation (14) is fairly intractable, and cannot be signed \textit{a priori}.

III.3 \textbf{Anticipated Future Tariff Changes}

We now consider the case of an anticipated change in future import tariffs. In order to focus the discussion we assume -- as we will do for the rest of the paper, unless otherwise indicated -- that initial tariff
levels are close to zero, and that there is gross substitutability in consumption everywhere. From (8) we can find the effects of anticipated tariffs on periods 1 and 2 equilibrium relative prices:

\[
\frac{dq^1}{dr^2} = - \frac{E^1_w}{\Delta} \left\{ E_{1q_2} \left( R_{q_2}^2 - E_{q_2}^2 \right) - E_{1q_1} \left( E_{p_2}^2 - R_{q_2}^2 \right) \right\} > 0 \quad (15)
\]

\[
\frac{dq^2}{dr^2} = - \frac{E^2_w}{\Delta} \left\{ (R_{q_2}^1 - E_{q_1}^1) (E_{p_2}^2 - R_{q_2}^2) + E_{1p_2} E_{q_2}^1 \right\} > 0 \quad (16)
\]

According to equation (15), if the public expects of the imposition of a future tariff, and as long as there is gross substitutability among M and N (i.e., \( E_{q_2}^{p_2} > 0 \)), there will be an appreciation of the equilibrium real exchange rate in the current period. Of course, the mechanism via which this takes place is the intertemporal substitution in consumption, captured in equation (15) by terms \( E_{1q_2}^p \) and \( E_{1q_2}^q \). If there is no intertemporal substitution, \( E_{1q_2}^p = 0 \) and \( E_{1q_2}^q = 0 \) then \( dq^1/dr^2 = 0 \). Equation (16) states that under our assumptions, an (anticipated) imposition of a period 2 tariff will also result in an equilibrium appreciation in that period. From an inspection of (15) and (16) it is apparent that it is not possible to know whether \( q \) will go up by more in period 1 or 2. Again, then, we have the possibility of "overshooting" or other "puzzling" time paths for the equilibrium RER.

III.4 Permanent Tariff Change and the Equilibrium Real Exchange Rate

If tariffs are changed permanently, then \( dr^1 = dr^2 \). Under the assumption of very small initial tariffs the effect on \( q^1 \) and \( q^2 \) will be given by:
\[
\frac{dq_1}{dr} = - \left( \frac{E_u}{\Delta} \right) \left\{ \left( E_{1,1} - R_{1,1}^1 + E_{1,2} \right)(R_{2,2}^2 - E_{2,2}^2)
+ E_{1,2} \left( E_{2,2}^{1,1} - R_{2,2}^2 + E_{2,2}^2 \right) \right\} > 0
\]

and

\[
\frac{dq_2}{dr} = - \left( \frac{E_u'}{\Delta'} \right) \left\{ \left( E_{1,1} - R_{1,1}^1 \right)(E_{2,1} - R_{2,2}^2 + E_{2,2}^2)
+ E_{2,1} \left( E_{2,2}^{1,1} - R_{2,2}^2 \right) \right\} > 0
\] (18)

Then, under gross substitutability in demand everywhere, both of these terms are positive, indicating that the imposition of a permanent tariff will result in a real appreciation in both periods. Whether this real appreciation will be larger in period 1 or in period 2, will depend on:

\[
\left( E_{1,1} - R_{1,1}^1 + E_{1,2} \right)(R_{2,2}^2 - E_{2,2}^2)
\geq \left( E_{1,1} - E_{1,1} \right)(E_{2,1} + E_{2,2} R_{2,2}^2).
\]

It is interesting to compare the reaction of the RER in period 1 for the cases of temporary and a permanent tariffs. From equations (17) and (11), and maintaining the assumption of gross substitutability, we find unequivocally that a permanent tariff will appreciate the equilibrium real exchange rate in period 1 by more than a temporary tariff imposed in that period only. In fact (17) can be rewritten as:

\[
\frac{dq_1}{dr} = \left( \frac{dq_{1,1}}{dr} \right) \text{Temp.} - \left( \frac{E_u}{\Delta} \right) \left\{ E_{1,2} \left( R_{2,2}^2 - E_{2,2}^2 \right) + E_{1,2} \left( E_{2,2}^2 - R_{2,2}^2 \right) \right\}
\] (19)
IV. Terms of Trade Shocks and the Equilibrium Real Exchange Rate

In this section we investigate how exogenous changes in international terms of trade \( (p^*_1 \text{ and } p^*_2) \) affect the equilibrium path of the real exchange rate. As in the preceding section the discussion will focus on three cases: (1) temporary terms of trade worsening (i.e., increase in \( p^*_1 \)); (2) anticipated future terms of trade worsening (increase in \( p^*_2 \)); and (3) permanent terms of trade worsening (equiproportional increase in \( p^*_1 \) and \( p^*_2 \)). Throughout the section we assume that initial tariffs are very low, so that we can evaluate our derivatives around \( r^1 = r^2 = 0 \).

IV.1 Temporary Terms of Trade Shocks

From (8), under the assumption that \( r^1 = r^2 = 0 \), we obtain that the vector of equilibrium RERs will react to a temporary increase in the international terms of trade \( (p^*_1) \) in the following way:

\[
\frac{dq^1}{dp^*_1} = - \frac{E_w}{\Delta} \left\{ \frac{(E^1 - R^1)}{q^1} \left( \frac{R^2}{q^2} - \frac{E^2}{q^2} \right) + \frac{E^2}{q^2} \frac{E^2}{q^2} \right\} + \left( \frac{1}{\Delta} \right) \frac{(E^1 - R^1)}{p^1} \left\{ \frac{E^2}{q^2} \frac{E^2}{q^2} \right\} \geq 0
\]

and,

\[
\frac{dq^2}{dp^*_1} = - \frac{E_w}{\Delta} \left\{ \frac{(E^1 - R^1)}{q^1} \left( \frac{R^2}{q^2} - \frac{E^2}{q^2} \right) + \frac{E^2}{q^2} \frac{E^2}{q^2} \right\} + \left( \frac{1}{\Delta} \right) \frac{(E^1 - R^1)}{p^1} \left\{ \frac{E^2}{q^2} \frac{E^2}{q^2} \right\} \geq 0
\]

A number of important results emerge from these equations. First, due to the existence of foreign borrowing a temporary terms of trade shock that increases the international price of importables today only, will affect both the current and future equilibrium value of the real exchange rate.
Second, contrary to the case of a temporary tariff, even under the assumption of gross substitutability everywhere, we now cannot sign these expressions. The reason for this is, of course, that in addition to the substitution effects, we now have a (negative) first order income effect associated to the worsening of the terms of trade. These income effects are given by the second RHS term in equations (20) and (21). As is usually the case they are proportional to the level of imports in period 1 \( (E_1 - R_1^*) \).

If the income effect dominates the substitution effect, \( (dq^1/dp^*) \) can be negative even if we assume gross substitutability in consumption everywhere. The reason, of course, is that the worsening of the terms of trade will result in a decline in demand for all goods in every period, generating a downward pressure on the relative price of nontradables in all periods.

In order to highlight the relation between tariffs and terms of trade effects, we can rewrite equation (20) in the following way (a corresponding expression can be written for equation (21)):

\[
\frac{dq^1}{dp^*} - \frac{dq^1}{d\theta} = \left( \frac{1}{\Delta_1} \right) \left( E_1 - R_1^* \right) \left\{ E_1 \pi_2 \pi_{q_2}^2 \pi_{q_2} \pi_{w_2}^1 + \pi_{q_1}^1 E_1 \left( R_2^2 - R_2^* \right) \pi_{q_2}^2 \right\}
\]

(22)

where, clearly the RHS of equation (22) is negative under our assumptions regarding substitutability in demand.

IV.2 **Anticipated Future Worsening in the Terms of Trade**

If in period 1 people anticipate that there will be a worsening of the terms of trade in the future, they will want to immediately adjust to it. The RER in period 1 will respond in the following way to an anticipated increase in the international price of imports:
\[
\frac{dq^{1}}{dp^{2}} - \frac{\left\{ E_{W}^{1} \right\}}{\Delta^{1}} \left( E_{q}^{1} p^{2} \left( R_{2}^{2} q^{2} - E_{2}^{2} q^{2} \right) + E_{2}^{2} p^{2} \left( R_{2}^{2} q^{2} - E_{2}^{2} q^{2} \right) \right)
\]

\[
+ \frac{1}{\Delta^{1}} \delta \star \left( E_{2}^{2} p^{2} - R_{2}^{2} \right) \left[ \pi_{q}^{2} E_{q}^{2} q^{2} \pi_{W}^{1} q^{2} + \pi_{q}^{1} E_{q}^{1} q^{2} \right]
\]

The first term on the RHS corresponds to the substitution effect and it is exactly the same as in the case of the imposition of a fully anticipated tariff (see equation 15 above). Under our maintained assumption of gross substitutability, it is positive. As before the mechanisms via which today's prices react to anticipated changes in future terms of trade are the intertemporal substitution effects \( E_{q}^{1} p^{2} \) and \( E_{q}^{2} p^{2} \); clearly, if there is no intertemporal substitution this first term will be equal to zero. The second term on the RHS is the discounted (negative) income effect generated by the future anticipated worsening of the terms of trade, and is proportional to the present value of imports in period 2. This second term is negative, generating forces towards a decline in equilibrium relative price of nontradables in the current period. The reason for this is that households try smooth consumption through time; the lower real income in the future, results in lower demand for \( N \) both in the present and in the future.

IV.3 Permanent Terms of Trade Shock

Equations (24) and (25) capture the effect of a permanent terms of trade shock on the vector of equilibrium RERs.

\[
\frac{dq^{1}}{dp^{2}} = \frac{dq^{1}}{dr} + \frac{1}{\Delta^{1}} \left( E_{q}^{2} q_{2} \pi_{q}^{2} E_{q}^{1} \pi_{W}^{1} q_{2} + \pi_{q}^{1} E_{q}^{1} \pi_{W}^{1} q_{2} \right)
\]

\[
\left( E_{q}^{1} - R_{2}^{1} \right) + \delta \star \left( E_{p}^{2} - R_{2}^{2} \right)
\]

\[
\frac{1}{p_{2}} + \frac{1}{p_{2}^{2}}
\]
\[
\frac{dq^2}{dp^1} = \frac{dq^2}{dr} + \frac{1}{\Delta_1} \left( \frac{(R^1_1 - E^1_1)}{q^1_1 q^1_1 \pi^1_1 \pi^2} \right) E^2_1 + E^2_1 \frac{\pi^1_1}{q^1_1 \pi^1_1 \pi^2} \\
\left[ (E^1_1 - R^1_1) + \delta^x (E^2_1 - R^2_1) \right] \\
\]  

where \(\frac{dq^1}{dr}\) and \(\frac{dq^2}{dr}\) are the pure substitution effects, and are given by equations (17) and (18) above. Under our simplifying assumptions of substitutability, these are positive. Notice that now the negative income effects are proportional to the present value of total imports. Again, as in the case of tariffs it is not possible to know a priori which relative price of nontradables will be affected by more as a result of a permanent terms of trade shock.

V. Terms of Trade Shocks and the Current Account

More than thirty-five years ago Laursen and Metzler (1950) and Harberger (1950) established conditions under which terms of trade shocks would worsen the current account, using essentially static models. More recently, Obstfeld (1982) Svensson and Razin (1983), and Persson and Svensson (1985) have relooked at the relation between terms of trade shocks and the current account using models where intertemporal considerations are explicitly taken into account. The specific question asked in these papers was: since the current account is equal to the difference between savings and investment, what are the mechanisms through which a terms of trade shock will affect these intertemporal decisions? Neither of these pieces, however, considered the case of home goods, where terms of trade shocks can have additional effects via changes in the RER.

Equations (26) and (27) provide expressions for changes in period 1's current account as a result of temporary and permanent terms of trade shocks.
(these assume $r_1 - r^2 = 0$):

\[
\frac{dCA^1_{\pi^1}}{dp^{\pi^1}_{\pi^1}} = -\pi^1 E^1 \pi^1 p^1 - \pi^1 E^1 \pi^1 q^1 \left(\frac{d\nu^1_{\pi^1}}{dp^{\pi^1}_{\pi^1}} \right)
\]

\[\text{temporary}\]

\[-\delta^* \pi^2 q^2 E^1 \pi^2 \pi^1 \left(\frac{d\nu^2_{\pi^1}}{dp^{\pi^1}_{\pi^1}} \right) - \left( E^1 - R^1 \right) - E^1 \pi^1 \left(\frac{dW^1}{dp^{\pi^1}_{\pi^1}} \right)\]

(26)

\[
\frac{dCA^1_{\pi^1}}{dp^{\pi^1}_{\pi^1}} = \left(\frac{dCA^1_{\pi^1}}{dp^{\pi^1}_{\pi^1}} \right) - \left( \delta^* E^1 \pi^2 \pi^1 p^2 \right)
\]

(27)

It is clear from equation (26), that in the present model it is not possible to know with certainty whether a temporary worsening in the terms of trade will improve or worsen the current account. The first three RHS terms of equation (26) are equivalent to those in equation (14) for the temporary tariff case, and their economic interpretation is virtually the same. The fourth RHS term in (26) is equal to period 1 imports and since it is preceded by a minus sign, it is negative. The last RHS term in (26) captures the (negative) income effect generated by a deterioration of the terms of trade, and is positive since $(dW/d\pi^1_{\pi^1}) < 0$ (i.e., the negative terms of trade shock reduces aggregate utility and real income.) These last two terms capture the fact that since as a result of the negative terms of trade shock, the country is poorer, expenditure will go down both in periods 1 and 2, generating forces towards improving the current account in period 1.

Equation (27) provides the response of the current account in period 1 to a permanent terms of trade shock. Notice that as before this expression cannot be signed unequivocally. In this model, even if there is a permanent terms of trade shock we cannot know \textit{a priori} whether the first period current account will improve or worsen. What we do know, from (27),
however, is that whatever the sign is, it will be smaller than in the case of a temporary shock only. The reason for this, of course, is that
\[-(\delta \pi^1 \pi^2 p^2_2)\] is negative. That is, a permanent negative terms of trade shock will either worsen the current account by more, or improve it by less, than a temporary shock. The reason for this is that when the terms of trade shock is permanent, the negative income effect affects both periods, and there is no intertemporal substitution of expenditure for consumption smoothing reasons.

Equation (28) provides the response of period 1's current account to an anticipated future deterioration of the terms of trade in period 2, and again cannot be signed a priori:

\[
\frac{dCA^1}{dp^{*2}} = -\delta \pi^1 \pi^2 p^2 - \pi^1 \pi^2 \pi^1 \pi^1 \left(\frac{dq^1}{dp^{*2}}\right) - \delta \pi^2 \pi^1 \pi^1 \left(\frac{dq^2}{dp^{*2}}\right)
\]

\[-\left(\pi^1 \pi^1 \pi^1 \left(\frac{dw}{dp^{*2}}\right)\right)\]  

VI. Extensions

The model derived above can be extended easily to handle a number of important questions related to the reaction of the equilibrium RERs and of the current account to different shocks. In this section we sketch some of the possible extensions of the model.

VI.1 Transfers

The model easily captures the effects of transfers from abroad on the vector of equilibrium real exchange rates. A transfer, denoted by H, in period 1 will have the following effect on \(q^1\) and \(q^2\), in the more general case of nonzero initial tariffs:
\[
\frac{dq}{dH} = -(\frac{1}{\Delta}) \left( E_{q_1} q_2^2 \pi_2^2 p_2 w + (R_{q_2}^2 q_2^2 - E_{q_2}^2 q_2^2) \pi_1^1 p_1 w \right) > 0 \tag{29}
\]

\[
\frac{dq}{dH} = -(\frac{1}{\Delta}) \left( E_{q_1} q_1^2 \pi_2^2 p_2 w + (R_{q_2}^2 q_2^2 - E_{q_2}^2 q_2^2) \pi_1^1 p_1 w \right) > 0 \tag{30}
\]

That is a positive temporary transfer from abroad will unequivocally appreciate the real exchange rate in both periods. The reason, of course, is that the transfer will result in higher present value of real income, a fraction of which will be spent in nontradables in each period, exercising upward pressure on their relative prices. Notice that if the propensities to spend on nontradables in both periods are zero (i.e., \( \pi_2^2 E_{q_2} - E_{q_2}^2 \pi_2 \pi_1 = 0 \)) then the transfer will have no effect on the real exchange rate in either period. The case of an anticipated future transfer is straightforward, and can be easily shown that it will also appreciate the current and future RER. Although transfers will result in an equilibrium real appreciation, in the present model they will always be welfare improving.\textsuperscript{13}

V.2 Factor Price Rigidities

All of the exercises performed above have assumed that all prices, including those of factors, are fully flexible. This is not always the case, especially in the developing countries. Rigidities in some factor prices can be easily introduced into the analysis. Assume, for example, that the (real) wage rate \( \bar{w} \) is fixed at a level \( \bar{w} \geq R_L \), where \( R \) is the unconstrained revenue function, and \( L \) is the labor force. In this case, then, we have to define a constrained revenue function \( R(\bar{w}, p, q, K) \) (Neary 1985):

\[
R(\bar{w}, p, q, K) = \max_{Q, L} \left( (Q^X + qQ^N + pQ^N) - \bar{w}L \right), \tag{31}
\]
where \( Q^i, i = X, M, N \) refers to output of exportables, importables and nontradables. Also, the nontradable market equilibrium conditions are replaced by:

\[
\begin{align*}
Q_1^i &= E_1^i; \quad Q_2^i = E_2^i \\
R_1^i &= q_1^i; \quad R_2^i = q_2^i
\end{align*}
\]  \( (32) \)

where \( \frac{\partial R^i}{\partial q} \) is the partial derivative of the constrained revenue function (31) with respect to the price of nontradables in period \( i \). Neary (1985) has shown that under fixed factor prices the following relation exists between restricted and unrestricted revenue functions:

\[
\tilde{R} = R[q, p, \tilde{L}(\hat{w}, q, p, K)] - \hat{w}L(\hat{w}, q, p, K)
\]  \( (33) \)

where \( \tilde{L} \) is the amount of labor employed in the constrained case. Once the revenue functions have been redefined in this way it is easy to find how the relative price of nontradables reacts to a tariff reduction in an economy with fix real wages.

VI.3 Tariffs, Real Exchange Rates and Employment

For a number of years trade theorists have been preoccupied with the relation between tariffs and employment (Mundell 1963; Eichengreen 1981; Kimbrough 1984; van Wijnbergen 1986). In the model developed in this paper, if wages are flexible, tariffs have no effects on aggregate employment. However, if there is real wage rigidity of the type described in Section VI.2 above, tariffs will indeed have an effect on the level of total employment in the economy. For example, equation (34) gives the response of labor employed in period 1 to a temporary tariff in that period.

\[
\frac{dL_1}{dr_1} = -\left(\frac{R_1^1}{L_q^1 R_1^1 / L_p^1 LL}\right) - \left(\frac{R_1^1}{L_q^1 R_1^1 / LL}\right) \frac{dq_1}{dr_1}
\]  \( (34) \)
where the term \( \frac{dq^1}{dr^1} \) captures the change in the relative price of \( N \) in period 1 to tariff increase. Both \( R^1_{LP} \) and \( R^1_{LQ} \) are Rybczinski type terms whose signs will depend on factor intensities. Depending on the sign of \( \frac{dq^1}{dr^1} \) and on factor intensities in the different sectors \( \frac{dL^1}{dr^1} \) can be positive or negative.

VI.4 Welfare Effects of Temporal Trade Liberalization

One of the advantages of the intertemporal framework based on duality theory developed in this paper is that it can be readily used to investigate the effects on welfare of alternative policies or exogenous disturbances. A particularly interesting question relates to the welfare effects of temporal trade liberalization. Economists have argued, for many years, that developing countries should reduce import tariffs and become more integrated to the rest of the world. Moreover, in the recent years a number of poor countries have in fact pursued policies aimed at reducing barriers to international trade. Many of these liberalization attempts, however, have been only temporary. For diverse reasons, after some time with lower tariffs, the liberalization reform is reversed, with tariffs being hiked once again to their old level. The question, then, is whether these temporary trade liberalizations are in fact welfare improving. This question can be easily answered using the system depicted in (8) in Section 2 above. It is easy to show that if \( r^2 > 0 \) a reduction in \( r^1 \) only may, under plausible conditions, be welfare reducing (i.e., \( dW/dr^1 < 0 \)). This suggests that if the possibilities of a reversal of the trade reform are high, it may not be convenient to attempt a temporary tariff reduction.
VI.5 Intermediate Inputs and Import Quotas

The intertemporal duality approach used here can be easily extended in order to incorporate import quotas and intermediate inputs. First, the case import quotas can be analyzed in a quite straightforward fashion by defining "virtual prices" as in Neary and Roberts (1980). The use of virtual prices, of course, assumes that the quota is allocated competitively via an auction mechanism.

Intermediate goods can also be incorporated quite easily through the definition of net-outputs as in Dixit and Norman (1980). In this case an additional source of ambiguity with respect to the sign of $dq/dr$ emerges.

VI.6 Investment

Since the discussion presented above has ignored investment, the current account in each period is equal to savings in that particular period. Investment, however, can be introduced in a straightforward fashion. Moreover, its incorporation will not alter in a significant way the main results presented above. Once investment is added to the analysis, the intertemporal budget constraint has to be altered and an equation describing the process governing investment decisions has to be added to our system. Denoting investment by $I$ and assuming that there is time to build, the intertemporal budget constraint becomes (where $V^1$ is the vector of factors of production other than capital):

$$
R^1(1,q^1,p^1;K^1,V^1) + \delta R^2(1,p^2,q^2;K^1+I,V^1)
+ r^1(E^1 - \frac{R^1}{p^1}) + \delta r^2(E^2 - \frac{R^2}{p^2}) - I(\delta*) = \frac{E[\pi^1(1,p^1,q^1),\delta\pi^2(1,p^2,q^2),w]}{}
$$

(35)
Further assuming that investment decisions are governed by the condition that in equilibrium Tobin's "q" equals 1, and that investment goods correspond to the numeraire good, the investment equation is:

\[ \delta R^2_k = 1 \]  

(36)

The manipulation of (35), (36) and the two conditions for equilibrium in the nontraded goods market in period 1 and 2 (equations (2) and (3)), will now yield the corresponding expressions for changes in the RERs and the capital account.

VI.7 World Interest Rates and Exchange Controls

The model of the preceding sections assumes that there is perfect capital mobility. However it can be easily amended to incorporate the case where capital flows are taxed. If this tax is prohibitive and there is no foreign borrowing, the model requires a period-by-period equilibrium in both the tradables and nontradables sector. This model has been discussed in detail by Edwards (1987).

The case of a non-prohibitive tax on capital movements can also be incorporated into the analysis. Under these circumstances the domestic discount factor is \( \delta < \delta^* \), and (the present value of) the tax on foreign borrowing per unit borrowed is equal to \( b = (\delta^* - \delta) (r - r^*) / (1 + r) (1 + r^*) \). In this case the intertemporal budget constraint has to be modified in two ways. First, \( \delta^* \) has to be replaced by \( \delta \). Second, under the assumption that the proceeds from this tax on foreign borrowing are returned to the private sector in a nondistortionary way, a tax proceeds term \( b (R^2 - \pi^2 \delta \pi^2) \) has to be added to the RHS of equation (1).

An interesting exercise is to investigate the form in which a change in the tax on foreign borrowing \( b \) affects the equilibrium RER. Edwards
(1987) has in fact shown that with no investment an increase in $\delta$ -- that is a liberalization of the capital account -- will always result in a real appreciation in period 1.

VI.8 Fiscal Deficits and the Equilibrium Real Exchange Rate

The analysis presented above has ignored the government sector. However, a policy question that has become increasingly important in the last year or so relates to the role of changes in fiscal policy on the equilibrium path of RERs. The model in this paper can be extended easily to analyze the role of fiscal policy. This, of course, will require adding the government budget constraint. Perhaps one of the more convenient ways to proceed is by assuming that the government finances its expenditures both by using the proceeds from the import tariffs and by borrowing from abroad. On the expenditure side it can be assumed that the government consumes both importables and nontradables. If the government consumption of tradables and nontradables in period 1 are denoted by $c^1_M$ and $c^1_N$, its (intertemporal) budget constraint is written as:

$$r^1 \left( R^1_p - E^1_p \right) + \delta r^2 \left( R^2_p - E^2_p \right) = p^1 G^1_M + q^1 G^1_N + \delta (p^2 G^2_M + q^2 G^2_N)$$

(37)

where the LHS is the discounted value of income from taxation and the RHS is the present value of government expenditure. After amending the private sector budget constraint and the equilibrium conditions in the nontradable markets, it is straightforward to find how changes in composition, size and financing of government consumption affects the RER and the current account. Notice, however, that in this case both tariffs and all of the government consumption cannot be exogenous. Now, in order to assure that (37) holds, $r^2$ has to be endogenous.
VII. Concluding Remarks

In this paper an optimizing intertemporal real model of a small open economy has been developed to investigate how various exogenous shocks affect the path of the equilibrium real exchange rate and the current account. It is assumed that firms produce competitively three goods -- exports, imports and nontradables. Households maximize the present value of utility, and consume all three goods. They have access to the international capital market, where they can borrow or lend at the given world interest rate. The only constraint they face is that the present value of the current account balances has to be zero. The model uses duality theory and exploits the properties of exact price indexes as developed by Svensson and Razin (1983).

The effects of both changes in import tariffs and of exogenous shocks to the international terms of trade were investigated, with emphasis placed on the distinction between temporary, permanent, anticipated and unanticipated disturbances. It was shown that in an intertemporal model with three goods a crucial channel through which exogenous shocks are transmitted is the consumption rate of interest (CRI). Changes in tariffs or the international terms of trade will affect the CRI, intertemporal expenditure decisions, and consequently the equilibrium vector of RERs and the current account.

It is shown that in the more general case -- where initial tariffs are high and where no restrictions are placed on the cross price derivatives in demand -- it is not possible to know how changes in tariffs or the terms of trade will affect the equilibrium path of the RER. This result is important, and contradicts the traditional policy oriented literature which claims that a hike in import tariffs result in a real appreciation. However, under
some (plausible) restrictions in the intertemporal model -- gross substitutability in demand everywhere and no initial tariffs -- it is possible to establish some important unambiguous results. The imposition of import tariffs -- either temporary or permanent -- will result in a real appreciation in both periods.

It is also shown that in this explicit general equilibrium intertemporal model with nontradable goods it is not possible to determine a priori the reaction of the current account to a tariff or a terms of trade change. Specific conditions for a current account improvement are derived. In the final section several directions in which the model can be extended are sketched, including the case of exchange controls, wage rigidity, and import quotas, and fiscal policies.
Appendix 1

Notation for Expressions in Equation 8

\[ E_{1\,1} = E_1 \pi_1 + E_1 \pi_1 \]
\[ E_{1\,1} = E_1 \pi_1 + E_1 \pi_1 \]
\[ E_{1\,2} = \pi_1 E_2 \pi_2 \]
\[ E_{1\,2} = \pi_1 E_2 \pi_2 \]
\[ E_{2\,2} = E_2 \pi_2 + E_2 \pi_2 \]
\[ E_{1\,1} = E_1 \pi_1 + E_1 \pi_1 \]
\[ E_{1\,1} = E_1 \pi_1 + E_1 \pi_1 \]
\[ E_{2\,2} = E_2 \pi_2 + E_2 \pi_2 \]
\[ E_{1\,2} = \pi_1 E_1 \pi_2 \]
\[ E_{1\,2} = \pi_1 E_1 \pi_2 \]
\[ E_{2\,2} = E_2 \pi_2 + E_2 \pi_2 \]
Appendix 2

Stability Conditions

In order to simplify the analysis of the stability conditions, and to sign the determinant, it is assumed that tariffs and the international terms of trade don't change. Thus, importables and exportables can be grouped into a composite called tradables. We denote the relative price of nontradables to tradables by $f$. 

The dynamic behavior of nontradable prices are depicted by equations (A.1) and (A.2), where $\lambda_1 \lambda_2 > 0$.

\[ \dot{f}^1 = \lambda_1 [E_{f^1} - R^1_{f^1}] \quad (A.1) \]

\[ \dot{f}^2 = \lambda_2 [E_{f^2} - R^2_{f^2}] \quad (A.2) \]

Using Taylor expansions of (A.1) and (A.2) around equilibrium prices, and dropping second and higher order terms, we obtain

\[
\begin{bmatrix}
\dot{f}^1 \\
\dot{f}^2
\end{bmatrix} = 
\begin{bmatrix}
\lambda (E_{\dot{f}^1 f^1} - R^1_{\dot{f}^1 f^1}) & \lambda_1 E_{f^1 f^2} \\
\lambda_2 E_{f^2 f^1} & \lambda_2 (E_{\dot{f}^2 f^2} - R^2_{\dot{f}^2 f^2})
\end{bmatrix}
\begin{bmatrix}
\dot{f}^1 - f^1* \\
\dot{f}^2 - f^2*
\end{bmatrix}
\]

Denoting the RHS matrix as $A$, stability of the system requires

\[ \text{Det } A > 0 \]

\[ \text{tr } A < 0 \]

This means that:

\[ \left\{ \left( E_{\dot{f}^1 f^1} - R^1_{\dot{f}^1 f^1} \right) \left( E_{\dot{f}^2 f^2} - R^2_{\dot{f}^2 f^2} \right) - E_{\dot{f}^2 f^1} E_{\dot{f}^1 f^2} \right\} > 0 \]
and
\[
\left\{ \left( E_{f\pi f}^{-1} - R_{f\pi f}^{-1} \right) + \left( E_{f\pi f}^{2} - R_{f\pi f}^{2} \right) \right\} < 0.
\]

These requirements can then be used to sign the determinant of the system in equation (8). Under the assumptions of this Appendix the matrix of the system in equation (8) is:

\[
B = \begin{pmatrix}
-(b\gamma f^2 E_{\pi f} f^1) & -b\delta E_{\pi f} f^2 & -(b^2 E_{\pi f}^2 + E_{\pi f}^0) \\
(R^1_{f\pi f} - E_{f\pi f}^1) & -\delta E_{f\pi f}^2 & -\pi f^1 E_{f\pi f}^1 \\
-E_{f\pi f}^2 & (R^2_{f\pi f} - E_{f\pi f}^2) & -\pi f^2 E_{f\pi f}^2
\end{pmatrix}
\]

where
\[
E_{f\pi f}^1 = (E_{\pi f}^1 + \pi f^1 E_{\pi f}^1)
\]
\[
E_{f\pi f}^2 = \pi f^1 E_{\pi f}^2
\]
\[
E_{f\pi f}^2 = (E_{\pi f}^2 + \pi f^2 E_{\pi f}^2)
\]

Using the stability conditions derived above it is possible to establish that the determinant of B is negative and equal to:

\[
\det B = \Delta'' = \left\{ -E_{f\pi f}^0 A - b\gamma f^2 E_{\pi f}^2 (R^1_{f\pi f} - E_{f\pi f}^1) R^2_{f\pi f} + b\gamma f^2 E_{\pi f}^2 (R^1_{f\pi f} - E_{f\pi f}^1) \right\} < 0.
\]
Footnotes

1 See, for example, Williamson (1983), Marris (1985).

2 There are some models that analyze the behavior of equilibrium RERs within a macroeconomic context. See, for example, Hooper and Roper (1982), and Mussa (1986).

3 Naturally, a model like the one presented here is too abstract to be directly applied to policy evaluation of RER behavior. It does provide, however, a number of important insights on the way exogenous shocks affect the equilibrium RER.

4 For a related model with exchange controls see Edwards (1987).


6 Notice that given the assumptions of our model, for this definition of equilibrium RER vector implies full employment.

7 Notice that \( \pi_1 E_{i_1} = C_{iE} E_{W_i} \), where \( C_{iE} \) is the marginal propensity to consume on imported goods in period 1 (Edwards and van Wijnbergen 1986).

8 Dornbusch (1980) and Corden (1985) assume no income effects in their static models. The negative sign of \( \Delta^1 \) follows from stability (see Appendix 2).

9 The intertemporal nature of this effect can be illustrated better by using the homogeneity property \( \pi_1 E_{i_1} + \pi_2 E_{i_2} = 0 \). to replace \( E_{i_1} \) in equation (14). Then this expression becomes:

\[
\frac{dCA^1}{dr^1} = \pi_1 E_{i_1} \pi_2 \left\{ 1 + \frac{\pi_1}{\pi_2} \left[ \frac{dq^1}{dr^1} - \frac{\delta \pi_2^1 dq^2}{\pi_2 dq^2} \right] \right\}
\]
Notice that if there is no intertemporal substitution, that is if \( E_{12} = 0 \), the imposition of a tariff has no effect whatsoever on the current account.

10 See Dornbusch (1980) for a good discussion on the more traditional models.

11 If the period specific substitutability functions are identical and there is no technological change these two terms will be equal, and
\[
\frac{dq_1}{dr} = \frac{dq_2}{dr}.
\]

12 The main difference is that now the indirect RER effect comes about via the impact of changes in \( p^1 \) on the vector of \( q \)'s.

13 In the small country case any transfer used for consumption purposes only are always welfare enhancing.

14 Perhaps the best examples of temporal trade liberalization are those of the Southern Cone of Latin America. See Edwards (1985) and Calvo (1986a). For theoretical analyses on the welfare effects of temporal trade reforms see Calvo (1985, 1986b).

15 Johnson (1966).

16 See Feldstein (1986) for a discussion of the relation between the U.S. fiscal deficit and the real value of the dollar. Frenkel and Razin (1986) have analyzed theoretically a number of issues related to fiscal policies in the world's economy.
References


