DURABLE GOOD MONOPOLY AND
BEST-PRICE PROVISIONS
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Working Paper No. 472
March 1988

*I would like to thank Bill Gale, Mike Waldman, and the participants of a workshop at UCLA for comments on an earlier draft of this paper.
ABSTRACT

Best-price provisions guarantee buyers that the price they pay is the lowest available. If the seller subsequently cuts price, then each previous buyer is entitled to a refund. It is shown here that a durable good monopolist who offers these provisions can construct a consistent plan which yields the same profits as if quantity precommitments were possible. The provisions are also effective if demand is uncertain or if the monopolist pursues objectives other than profits. Unlike some alternate means for resolving the monopolist’s commitment problem, these provisions work regardless of the good’s ex post specificity or the nature of uncertainty. They are less effective if the monopolist can discriminate using nonprice preferences or if refunds are costly to administer.
I. INTRODUCTION

In a classic (1972) paper, Ronald Coase conjectured that a monopoly seller of an infinitely durable good cannot credibly sell output at the static monopoly level. Once the initial quantity of output has been sold, the monopolist is tempted to sell additional amounts as long as price remains above marginal cost. Without some restraint on sales, the market will be saturated with the competitive output "in the twinkling of an eye."

Coase's reasoning has been supported by several subsequent authors, including Stokey (1981), Bulow (1982), Bond and Samuelson (1984), Gul, Sonnenschein, and Wilson (1986), and Kahn (1986). Although the results from these various works do not always suggest that the monopolist will behave competitively, the qualitative outcome is the same: the monopolist cannot reap the full monopoly rewards without some commitment to limit output.¹

While such actions cannot be ruled out empirically, firms with substantial market power do not often behave competitively.² This suggests that either real-world examples do not take the form of Coase's illustration, or monopolists precommit not to behave competitively.

Ausubel and Deneckere (1986) make a case for the first possibility. Initially, consumers believe they are facing a "strong" monopolist which will not deviate from its announced plan. The moment a deviation occurs, however, these consumers realize that they are dealing with a "weak" monopolist (which knows of the Coase Conjecture). The prospect of such an abrupt awakening disciplines the monopolist and prevents price from being cut. Ausubel and Deneckere show that as the time interval between successive periods approaches zero, the set of potential monopolist payoffs expands to include the entire interval from zero to static monopoly profits.
The second possibility has received greater attention. Coase offers three options for addressing this problem. First, the monopolist can precommit not to sell additional output. Second, the good can be rented rather than sold. Third, the monopolist can precommit to repurchase the good if a lower price is ever offered.

Other authors have extended this list of options. Credible commitments to restrict future output can also be made by limiting production capacity (Stokey (1981)) or by adopting technologies with increasing marginal cost (Bulow (1982)). The good's durability can be reduced (Bulow (1982)) or monopoly power can be transferred from the durable good to a related non-durable good (Bulow (1982)). These options may not completely resolve the monopolist's problem, but they all mitigate it to some degree.

This paper offers yet another option. In particular, the monopolist's problem can be resolved completely and costlessly through the use of best-price provisions. These provisions guarantee that the price to be paid is the lowest available. If better terms are subsequently negotiated in any related contract, then the monopolist must refund the difference between the original price and the new lower price. The clause appears in settings as diverse as retail trade, long-term supply contracts, and international commodity treaties.

After providing some background (Section II), a simple explanation (Section III) and formal model (Section IV) verify that for the durable good monopolist posited by Coase and others, best-price provisions yield a time consistent plan for reaping full monopoly profits. Cost and demand uncertainty are introduced in Section V. A numerical example (Section VI) illustrates the provision's mechanics. Section VII discusses the advantages and disadvantages of best-price provisions and a conclusion follows.
II. BACKGROUND ON BEST-PRICE PROVISIONS

Although best-price provisions are pervasive in many economic contexts, their scope is typically restricted. Retailers, for example, often extend the provision only to the same brand name and model, and limit it to specific time periods and geographic areas. "Three-party" best-price provisions (or "meet-the-competition" clauses) guarantee the lowest price offered by any seller of the good; "two-party" versions apply only to the lowest price offered by the seller involved in the original transaction.

International commodity agreements have employed best-tariff terms, known as "most-favored-nation" (MFN) provisions, for over three centuries. By offering such provisions, a country promises each trading partner access to its domestic markets at tariff rates which are no higher than those offered by that country to any other trading partner. The consensus in the international trade literature is that MFNs assure nondiscrimination:

... the most-favored-nation clause conferred no privileges of any great importance, for the general rule was to treat all nations as equals. Most-favored-nation treatment then, meant not favored treatment, but merely a guarantee against being less favorably treated than other foreign nations. (Setser (1937), page 69)

For over fifty years natural gas producers have also bargained for best-price provisions in their contracts with pipelines, and potential discrimination has been forwarded as one explanation for their use (Neuner (1960), Butz (1986), and Hubbard and Weiner (1986)).

Other authors (Scherer (1980), Grether and Plott (1981), Holt and Scheffman (1985), Salop (1986), and Cooper (1986)) argue that two-party best-price provisions may be effective mechanisms for tacit collusion. These provisions commit a firm to refund to existing customers the difference between their price and the prices paid by subsequent customers. This discourages aggressive competition for new customers. Belton (1986)
demonstrates how three-party best-price provisions can also enhance collusion by committing each firm to match the prices of all industry rivals.

Best-price provisions in field markets for natural gas have been forwarded by MacAvoy (1962), Broadman and Montgomery (1983), and Broadman and Toman (1984) as a means for sharing risks. In particular, the provisions shift the risks associated with price uncertainty from the beneficiary of the clause to the benefactor.

Attention here focuses on the nondiscrimination motive for best-price provisions. Both two- and three-party versions are modeled. Although no attempt is made to test this model here, such tests have been conducted elsewhere (see Butz (1986)).

III. A SIMPLE EXPLANATION OF BEST-PRICE PROVISIONS

A hypothetical demand function for ownership of a durable good is illustrated in Figure 1. The price of this good, \( P(X) \), is decreasing in the quantity \( X \) of the good sold. If a monopoly seller of this good produces at zero marginal cost, then the static monopoly solution entails selling \( M \) units at price \( P(M) \) per unit.

Coase's revelation is that this textbook solution does not hold in the context of a dynamic monopoly model. Having sold \( M \) units, the monopolist is tempted to lower price in order to sell additional output. Realizing this, prospective buyers balk at paying \( P(M) \). Unless the monopolist can commit not to cut price, no output can be sold at any price above marginal cost.

Now consider the outcome when best-price provisions are offered to all buyers. The monopolist begins by selling \( M \) units at a price of \( P(M) \) per unit. If \( \Delta M \) additional units are then sold, the value of each of the first
Figure 1: Demand for a Durable Good

M units falls by \( P(M) - P(M+\Delta M) \). Best-price provisions entitle the original buyers to rebates which offset this loss completely. In the same fashion, additional sales lower the market value of all units sold previously, but buyers always receive full compensation for their losses. An increase in output \( X \) costs the monopolist \( \Delta P \times X \) in rebates, but increases revenues by \( P \times \Delta X \). The monopolist sets \( X \) such that marginal rebates equal marginal revenue. This yields the static monopoly outcome.

Best-price provisions are similar to repurchase agreements. Suppose such agreements are used in lieu of best-price provisions in the previous example. The first \( M \) units again sell for \( P(M) \) and the \( \Delta M \) additional units sell for \( P(M+\Delta M) \). Once the additional \( \Delta M \) units are sold, the original \( M \) units can be redeemed by buyers for \( P(M) \) and then repurchased from the monopolist for \( P(M+\Delta M) \). Best-price provisions and repurchase agreements therefore differ in only one respect: while repurchase agreements require the monopolist to reassemble ownership of the good, best-price provisions do not. If the good has some ex post specificity, then best-price provisions
offer a clear advantage over repurchase agreements.

IV. THE FORMAL MODEL

The model developed here follows the framework outlined by Kahn (1986). At time \( t = 0 \) a (new) durable good is introduced. This good never depreciates and is available initially only through a monopoly seller. Once purchased, however, the good can be leased or sold through perfectly competitive secondary markets.

\( Q(t) \) is the total stock of the good at time \( t \), and \( Q'(t) \) equals the firm's flow of production. \( \phi(Q(t)) \) is the stationary inverse demand for the good's rental services and is known to all agents. The continuous discount factor \( r \) is common to the seller and to all buyers. With perfect secondary markets, the price of the good at time \( t' \) is given by

\[
(1) \quad P(t') = \int_{t'}^{\infty} \phi(Q(s)) e^{-r(s-t')} \, ds
\]

Production costs, \( \gamma(Q'(t)) \), are increasing and convex. If the firm can precommit at time \( t = 0 \) to a production plan \( (Q'(t), 0 \leq t) \), it chooses \( Q(.) \) to maximize discounted profits, \( \Pi \), where

\[
(2) \quad \Pi = \int_{0}^{\infty} (P(t)Q'(t) - \gamma(Q'(t))) e^{-rt} \, dt
\]

Let \( (Q^*(t), 0 \leq t) \) be the plan which maximizes \( \Pi \).

Without precommitment, however, this choice is not time consistent:

...suppose we were to stop the monopoly problem at some future date \( t_1 \) and allow the monopolist to reoptimize by picking the profit-maximizing plan \( \tilde{Q}(t) \) for the remaining time, subject to the constraint that \( \tilde{Q}(t_1) = Q^*(t_1) \) (so that what has been made available to date be regarded as the initial stock for the new problem). Then in general \( \tilde{Q}(t) \neq Q^*(t) \) for \( t > t_1 \) (Kahn, p. 280).
Now suppose the monopolist extends best-price provisions to all buyers from the outset. Let $\rho(t)$ be the price paid at time $t$ by a buyer receiving a best-price provision. Then at time $t'$

$$\rho(t') + \int_0^\infty \rho'(s)e^{-r(s-t')} \, dt = \int_{t'}^\infty \phi(Q(s))e^{-r(s-t')} \, ds$$

The left-hand-side of (3) equals the net cost of the good -- the purchase price minus the discounted value of all refunds. The right-hand-side is the discounted value of the future rental services. Integration by parts gives

$$\int_{t'}^\infty (r\pi(s)e^{-r(s-t')}) \, ds = \int_{t'}^\infty \phi(Q(s))e^{-r(s-t')} \, ds$$

By Leibnitz's rule, differentiation of (4) with respect to $t'$ gives

$$\rho(t') = (r^{-1})\phi(Q(t')).$$

Best-price provisions protect buyers from adverse changes in the good's asset value. Hence, $\rho(t)$ depends only on the current value of the good's rental services.

With best-price provisions, the monopolist's revenues at time $t$ equal the proceeds from sales of the good, $\rho(t)Q'(t)$, minus refunds made to former customers, $\rho'(t)Q(t)$. The monopolist chooses $Q(.)$ to maximize $\Omega$, where

$$\Omega = \int_0^\infty (\rho(t)Q'(t) + \rho'(t)Q(t) - \gamma(Q'(t)))e^{-rt} \, dt$$

The first result demonstrates that the monopolist's revenues are unaffected by best-price provisions.

**Lemma:** For any production plan $(Q'(t), 0 \leq t)$, discounted revenues are the same whether or not best-price provisions are used in place of simple prices.
Proof: See Appendix.

Since best-price provisions do not affect production costs, the first proposition follows immediately from the lemma.

Proposition 1: For any given production plan \((Q'(t), 0 \leq t)\), best-price provisions yield the same discounted profits as simple price provisions. In other words, \(\Omega = \Pi\).

Proof: See Appendix.

Even though best-price provisions do not alter profits, their advantage over simple price provisions is clear from the second proposition.

Proposition 2: If the monopolist adopts best-price provisions at the outset, then the production plan \((Q^*(t), 0 \leq t)\) is time consistent.

Proof: See Appendix.

The intuition parallels the simple example from Section III. By equation (5), a one-unit increase in output at time \(t\) yields \([\phi(t)/r]\) in additional revenues, but requires a refund of \([\phi'(t)/r]\) to all previous buyers. This refund fully compensates for the reduction this output causes in the value of units sold previously. The monopolist increases output until additional revenues minus refunds equal marginal cost. This gives the same Euler condition derived by choosing \(Q(\cdot)\) to maximize \(\Pi\).

Together, the first two propositions provide the paper’s main result. The first proposition demonstrates that best-price provisions are costless to employ; the second shows that they are completely effective in resolving the monopolist’s time inconsistency problem.

The results also hold when the monopolist pursues other objectives (e.g., when the monopolist’s output is set by a social welfare-maximizing
regulator). Suppose a customer purchases one unit of the good at time $t_1$ and receives a best-price provision. If the good is then resold at any future date $t_2$, then if follows directly from equation (5) that

$$
\rho(t_1) - e^{-r(t_2-t_1)} \rho(t_2) + \int_{t_1}^{t_2} \rho'(s)e^{-r(s-t_1)} ds - \int_{t_1}^{t_2} \phi(Q(s))e^{-r(s-t_1)} ds
$$

The purchase price minus the discounted sale price plus the discounted refunds sum to exactly the discounted value of the rental services consumed, regardless of actions taken by the monopolist after $t_1$. In short, best-price provisions assure buyers of nondiscrimination even in cases where monopoly objectives include considerations other than profits.

V. UNCERTAINTY AND NON-STATIONARY DEMAND

Best-price provisions compensate buyers for any losses caused by subsequent sales, thereby allowing the monopolist to construct a time consistent plan. With the introduction of uncertainty, however, the good can change in value for reasons unrelated to output levels, and best-price provisions are incapable of distinguishing between these two causes of price changes. One might think, therefore, that best-price provisions are ineffective except under conditions of perfect certainty. It is shown in this section that uncertainty does not reduce the effectiveness of these provisions as long as the monopolist is willing to assume the risk associated with this uncertainty.

For expositional purposes, a discrete-time framework is adopted.\textsuperscript{5} Otherwise, the model proceeds as before. $Q_t$ is the total stock of the good at time $t$ and $q_t = (Q_t - Q_{t-1})$ is the quantity sold by the monopolist. $\phi_t = \phi_t(Q_t, \epsilon_t)$ is the inverse demand for the good's rental services, where $\epsilon_t$ is
a random variable with probability density function $f_t(\epsilon_t)$. Production costs are $\gamma_t = \gamma_t(q_t)$, where $\gamma_t'(q_t) \geq 0$ and $\gamma_t''(q_t) \leq 0$. There is a common discount rate, $\delta$, and all agents know $\gamma_t$, $f_t$, and $\phi_t$.

Risk neutrality and perfect secondary markets imply

$$(7) \quad P_{t'} = \phi_{t',(Q_t',\epsilon_t')} + E_{t'} \sum_{t'+1}^{\infty} \delta^{(t-t')} \phi_t(Q_t,\epsilon_t)$$

It follows directly from (7) that

$$(8) \quad P_{t'} = \phi_{t',(Q_t',\epsilon_t')} + \delta E_{t'} P_{t'+1}$$

Profits are perfectly analogous to the continuous-time formulation:

$$(9) \quad \Pi = (\{q_0 - \gamma_0\}q_0) + \sum_{t=1}^{\infty} \delta^t (P_t q_t - \gamma_t(q_t))$$

At time 0, the monopolist chooses $(q_t, t=0,1,...)$ to maximize expected profits. The contingent plan maximizing $\Pi$ is given by $(q_t^*, t=0,1,...)$, where $q_t^*$ is contingent on all information available at time $t$. As before, this plan is not time consistent without precommitment.

A. INFINITE-DURATION, TWO-PARTY BEST-PRICE PROVISIONS

One means for resolving the commitment problem is to offer two-party best-price provisions which extend forever but are limited in scope -- they apply only to subsequent transactions involving the monopolist. Let $\rho_t$ be the monopolist’s time-$t$ price when these provisions are offered. The clause provides each buyer with a rebate at time $t+k$ equal to $(\rho_{t+k-1} - \rho_{t+k})$.

The net undiscounted cost to the time-$t$ buyer at time $t+k$ is $\rho_{t+k}$. The net discounted cost of owning the good is given by the left-hand-side of equation (10):
\( \pi_t' - E_t \sum_{t'+1}^{\infty} \delta^{(t-t')} (\pi_{t-1} - \rho) = \phi_t'(Q_t', \epsilon_t) + E_t \sum_{t'+1}^{\infty} \delta^{(t-t')} \phi_t(Q_t', \epsilon_t) \)

The right-hand-side is the expected value of the good's rental services.

It is straightforward to show that

\( \rho_{t'} = (1 - \delta)^{-1} \phi_t'(Q_t', \epsilon_t) \)

Because the monopolist assumes the consequences of any change in price, buyers are unconcerned about future events. As in the perfect certainty setting, \( \rho_{t'} \) is set as if inverse demand will forever equal \( \phi_t' \).

Profits using these two-party best-price provisions are given by \( \Omega \):

\( \Omega = (\rho_0 q_0 - \gamma_0(q_0)) \sum_{t=1}^{\infty} \delta^t (\rho_{t-1} q_t - (\rho_{t-1} - \rho_t) Q_{t-1} - \gamma_t(q_t)) \)

The impact of these provisions on monopoly profits is considered shortly. Before proceeding to these results, a second option is outlined.

B. ONE-PERIOD MEET-THE-COMPETITION CLAUSES

The time inconsistency problem can also be resolved with one-period meet-the-competition (MTC) clauses. Let \( \alpha_t \) be the time-\( t \) price paid by buyers when these provisions are offered. The clauses provide each buyer with a rebate at time \( t+1 \) equal to \( (\alpha_t - P_{t+1}) \), where \( P_{t+1} \), defined by equation (7), is the secondary market price at time \( t+1 \). Hence,

\( \alpha_t' - \delta E_t'(\alpha_{t'} - P_{t'+1}) = \phi_t'(Q_t', \epsilon_t) + E_t \sum_{t'+1}^{\infty} \delta^{(t-t')} \phi_t(Q_t', \epsilon_t) \)

The left- and right-hand-sides of (13) are, respectively, the buyer's expected discounted payments and the discounted value of rental services.

It is straightforward to show the analog to equations (5) and (11):
\[ (14) \quad \alpha_t = (1 - \delta)^{-1} \phi_t(Q_t, \epsilon_t) \]

Monopoly profits using MTC clauses are given by \( \Gamma \), where

\[ (15) \quad \Gamma = (\alpha_0 q_0 - \gamma_0(q_0)) + \sum_{t=1}^{\infty} \delta^t (\alpha_t q_t - (\alpha_{t-1} - p_t)q_{t-1} - \gamma_t(q_t)) \]

C. TIME CONSISTENCY PROPOSITIONS

The next two results, the analogs to Propositions 1 and 2, demonstrate that the monopolist at time 0 can generate a time consistent plan yielding the same profits as if precommitment were possible.

Proposition 3: For any given plan \( (q_t, t=0,1,\ldots) \), either of these best-price provisions yields the same expected discounted profits as simple price provisions. In other words, \( E_0 \Omega = E_0 \Gamma = E_0 \Pi \).

Proof: See Appendix.

Proposition 4: By offering either of the best-price provisions defined above, the monopolist's choice of \( (q_t^*, t=0,1,\ldots) \) becomes time consistent.

Proof: See Appendix.

VI. A NUMERICAL EXAMPLE

A hypothetical event is being funded in part through sales of a commemorative lithograph. There is only one seller of these prints, the event's sponsor, who produces at zero marginal cost and maximizes expected profits. Sales take place before \( (t=0) \) and after \( (t=1) \) the event. The sponsor can credibly commit to destroy the lithograph at the end of the second period, so there are only two production decisions, \( q_0 \) and \( q_1 \). The prints are infinitely durable and the interest rate is five percent.
At time $t=0$, 1000 individuals are prepared to pay up to $100 to own the print (or five dollars per period to rent it). At time $t=1$, an additional $\eta_1$ individuals will be willing to pay up to $60 to purchase the print, but $\eta_1$ is not known at time $t=0$. For simplicity, assume that all parties believe that $\eta_1$ is distributed uniformly over the interval $[0, 1000]$. The demand for ownership at time $t=1$ is given in Figure 2.

Three possibilities are considered here: simple prices with quantity precommitments (Case 1), simple prices without quantity precommitments (Case 2), and best-price provisions (Case 3). Table 1 summarizes the results.

**Case 1**: Quantity precommitments are allowed. 1000 units are sold at time $t=0$. Additional sales at time $t=1$ generate revenues of $60$ times $\eta_1$, but reduce the value of prints sold at time $t=0$ by $40$ times 1000. Hence, the monopolist precommits at time $t=0$ to refrain from additional sales at time $t=1$ unless $\eta_1 > 666$. Under these terms, $P_0 = 87.50$ and expected profits equal $103,413$.

**Case 2**: Quantity precommitments are prohibited. If 1000 units are sold at time $t=0$, then the monopolist always cuts price at time $t=1$. Knowing
| Case 1 (Precommitment) | 1000 | 87.54 | 666 | 103,413 |
| Case 2 (No Precommitment) | 197 | 82.54 | 535 | 98,902 |
| Case 3 (Best-Price) | 1000 | 100.00 | 666 | 103,413 |

Table 1: Profit-Maximization Under Three Scenarios

this, time-0 buyers adjust their demand accordingly. The monopolist’s only recourse is to restrict output at the outset. Expected profits are maximized by setting \( q_0 = 167 \). At time \( t=1 \), price is cut if and only if \( \eta_1 > 535 \). Under this plan, \( P_0 = 82.50 \) and expected profits equal \$98,902.

**Case 3:** Best-price provisions are employed. Buyers at time \( t=0 \) are unconcerned about price at time \( t=1 \). The monopolist sets \( q_0 = 1000 \). At time \( t=1 \), the monopolist weighs the revenues from additional sales of the good \( (\$60 \text{ times } \eta_1) \) against the rebates that would have to be paid to previous buyers \( (\$40 \text{ times } 10000) \). Price is cut if and only if \( \eta_1 > 666 \). Because best-price provisions redistribute risk from time-0 buyers to the monopolist, the initial price and the realized profits are different than in the precommitment case. Yet output and expected profits are the same.

As a fourth case, suppose the monopolist attempts to address the time inconsistency problem by indexing \( P_0 \) directly to \( \eta_1 \) (e.g., buyers receive a \$40 refund whenever \( \eta_1 > 666 \)). This method of indexing price is independent of the monopolist’s actions, and does **nothing** to resolve the problem. If
q_0 = 1000, the monopolist always cuts price at time t-1. In short, standard price indexes do not replicate the outcome of best-price provisions.

VII. RELATIVE MERITS OF BEST-PRICE PROVISIONS

The numerical example of the previous section assumes that the monopolist can credibly commit to destroy the lithograph, but cannot commit, at least in two of the cases, to limit production. Since artists routinely offer "limited editions" of their work, quantity precommitments must be considered a legitimate mechanism for addressing the time inconsistency problem. Yet in theory these precommitments are neither more nor less effective than best-price provisions, rental agreements, or repurchase provisions. Why do artists prefer such limited editions over other options? Why do international trade agreements in all cases opt for most-favored-nation provisions? This section addresses these questions by outlining some of the costs and benefits associated with best-price provisions.

Some of the advantages of best-price provisions have already been mentioned. Unlike rental or repurchase agreements, best-price provisions do not require the monopolist to reassemble ownership. Hence, they are relatively more attractive when the good has some ex post specificity. Best-price provisions also work regardless of the monopolist's intentions. In fact, neither the monopolist nor the buyers need to know what these future intentions will be. Finally, best-price provisions may (or may not) redistribute risk in an efficient manner.

The administrative costs surrounding best-price provisions may also be low. Although future prices must be observable, other information costs are negligible. Compare this to quantity precommitments which restrict the monopolist's actions far into the future and require buyers to monitor
behavior over long periods of time. Where uncertainty is present, conditional quantity precommitments may be difficult to construct ex ante and to enforce ex post. In contrast, the administrative costs of best-price provisions are largely unrelated to the nature of uncertainty.

Several means for resolving the monopolist's time inconsistency problem permit limited pursuit of intratemporal discrimination. Best-price provisions, for example, can be limited to specific geographic areas, brand names, or models. While these provisions do not necessarily afford greater or lesser opportunities for discrimination than other options, such prospects may be a consideration when choosing between alternatives.

Best-price provisions are not without their disadvantages. If prices are falling over time, then two-party versions of the clause require recurring refunds. If prices are expected to rise at any point, then the monopolist faces a difficult choice: to require buyers to make additional payments at that time or to forego the income from such price increases. Best-price provisions have the greatest appeal, therefore, where subsequent payments are either unlikely or inexpensive to distribute.

Several means for resolving the monopolist's time inconsistency problem permit limited pursuit of intertemporal discrimination (which may be profitable, for example, if individuals have differing discount rates and secondary markets are imperfect). Best-price provisions do not afford such possibilities. Hence, the monopolist may choose to employ other forms of commitment in such circumstances.

Because best-price provisions do not preclude intertemporal quality discrimination, they are less effective when the durable good is heterogeneous. This problem can be addressed by modifying the provisions to account for product differences. For example, natural gas contracts sometimes
promise the most favorable quality-adjusted price, and are often specified net of transportation costs. The effectiveness of best-price provisions in such situations can also be enhanced by coupling them with guarantees of most favorable treatment along other economically relevant dimensions of the contract. Nonetheless, heterogeneity reduces the attractiveness of best-price provisions relative to other options for resolving the monopolist's problem.

VIII. CONCLUSIONS

Although best-price provisions require special circumstances to be practical, they are remarkably simple and effective, and they appear in a wide variety of contexts. These include retail trade, long-term supply agreements, international trade, and loan contracts indexed to banks' prime lending rates.

Variations of these provisions also appear in less obvious settings. Consider a firm with a market value of $V$ which sells $N$ shares of stock in an initial public offering. There is a single seller of the stock, the owner-manager, and the equity sold represents a durable claim on future profits.

Subsequent sales of equity will dilute each shareholder's proportionate claim on the firm. Each shareholder is compensated for this dilution, since the proceeds of the sale are retained by the firm. If the sale is nondiscriminatory, then it is straightforward to show that this compensation exactly offsets the reduction in each share's proportionate claim on the firm. But if the stock is sold at a price below $V/N$ (or if stock is repurchased at a price above $V/N$), then the proceeds from the sale are insufficient to compensate existing shareholders.
Conventional criticisms of shareholder discrimination have typically been founded on normative grounds. This paradigm offers a positive critique: firms which do not follow a policy of nondiscrimination face a higher cost of capital than those which do. Since "equal treatment" provisions appear either explicitly or implicitly in a variety of contracts, similar arguments could be made in other contexts.

While the assumptions made here lead to contractual guarantees of nondiscrimination, in practice many contracts do precisely the opposite. Seniority and rights-of-first-refusal are just two examples of contract provisions which explicitly discriminate intertemporally. A second area for future research entails a more general model of such discrimination.

Best-price provisions also have interesting welfare implications. Simply put, they permit the durable good monopolist to behave like a monopolist rather than a competitor. Yet as the shareholder example illustrates, their appearance can be welfare-enhancing. A third research project seeks to investigate such issues in greater detail.

Finally, the model here must be tested empirically against such competing explanations for best-price provisions as collusion among oligopolistic sellers and risk-sharing. Butz (1986) has conducted such tests using field market contracts for natural gas, and the evidence in this context supports the nondiscrimination motive for best-price provisions. Other settings should also be explored to determine whether or not other proposed motivations are involved.
**APPENDIX**

**Lemma:** For any production plan \( (Q'(t), 0 \leq t) \), discounted revenues are the same whether or not best-price provisions are used in place of simple prices.

**Proof:** Formally, we wish to show that
\[
\int_{0}^{\infty} \left( \rho(t)Q'(t) + \rho'(t)Q(t) \right)e^{-rt} dt
= \int_{0}^{\infty} (P(t)Q'(t))e^{-rt} dt - \int_{0}^{\infty} \phi(Q(t))Q(t)e^{-rt} dt.
\]
Let \( u = e^{-rt} \) and let \( v = \rho(t)Q(t) \). Integration by parts gives
\[
\left( \int_{0}^{\infty} (\rho(t)'(t) + \rho'(t)Q(t))e^{-rt} dt \right)_{0}^{\infty}
= \left( \int_{0}^{\infty} \rho(t)Q(t)e^{-rt} dt \right)_{0}^{\infty}.
\]

By equation (3),
\[
(r) \int_{0}^{\infty} \rho(t)Q(t)e^{-rt} dt = (r) \int_{0}^{\infty} (r^{-1})\phi(Q(t))Q(t)e^{-rt} dt.
\]
and simplifying gives
\[
\int_{0}^{\infty} \left( \rho(t)Q'(t) + \rho'(t)Q(t) \right)e^{-rt} dt
= \int_{0}^{\infty} \phi(Q(t))Q(t)e^{-rt} dt - \int_{0}^{\infty} \phi(Q(t))Q(t)e^{-rt} dt.
\]

Now let \( u = P(t)e^{-rt} \) and let \( v = Q(t) \). Then integrating by parts gives
\[
(A1) \int_{0}^{\infty} (P(t)Q'(t))e^{-rt} dt
= \int_{0}^{\infty} Q(t)(P(t)(-r) + P'(t))e^{-rt} dt.
\]

Applying Leibnitz's Rule to equation (1) reveals that
\[
(A2) P'(t) = -\phi(Q(t)) + (r) \int_{t}^{\infty} \phi(Q(s))e^{-r(s-t)} dt - \phi(Q(t)) + (r)P(t).
\]

Substituting (A2) into (A1) gives
\[
\int_0^\infty (P(t)Q'(t))e^{-rt} \, dt = \int_0^\infty \phi(Q(t))Q(t)e^{-rt} \, dt. \quad \text{Q.E.D.}
\]

**Proposition 1:** For any given production plan \((Q'(t), 0 \leq t)\), best-price provisions yield the same discounted profits as simple price provisions. In other words, \(\Omega = \Pi\).

**Proof:** From the lemma, discounted revenues are the same under both plans. Production is also identical. Hence, discounted profits are equal. Q.E.D.

**Proposition 2:** If the monopolist adopts best-price provisions at the outset, then the production plan \((Q^*(t), 0 \leq t)\) is time consistent.

**Proof:** Define \(\Omega(a,b)\) as follows:

\[
(A3) \quad \Omega(a,b) = \int_a^b \left( \rho(t)Q'(t) - \rho'(t)Q(t) - \gamma(Q'(t)) \right)e^{-rt} \, dt
\]

The monopolist chooses \((Q'(t), t_1 \leq t)\) at time \(t_1\) to maximize \(\Omega(t_1, \infty)\) subject to \(Q(t_1) = Q^*(t_1)\). Through integration by parts \((u = e^{-rt}; v = \rho(t)Q(t))\) and use of equation (5), it can be shown that for any \(a\) and \(b\),

\[
(A4) \quad \Omega(a,b) = \left( r^{-1} \phi(Q(t))Q(t)e^{-rt} \right)_{a}^{b} + \int_a^b (\phi(Q(t))Q(t) - \gamma(Q'(t)))e^{-rt} \, dt.
\]

Note that \(\Omega(a,b)\) is independent of any actions taken either before time \(a\) or after time \(b\). Let \(\Omega^*(a,b)\) refer to \(\Omega(a,b)\) when the monopolist follows the plan \((Q^*(t), 0 \leq t)\) from time \(a\) to time \(b\). Similarly, let \(\tilde{\Omega}(a,b)\) refer to \(\Omega(a,b)\) when the monopolist follows plan \((Q'(t), 0 \leq t)\). By (9) and (A14),

\[
(A5) \quad \Omega^* - \Omega^*(0, \infty) = \Omega^*(0, t_1) + \Omega^*(t_1, \infty) \geq \Omega^*(0, t_1) + \tilde{\Omega}(t_1, \infty)
\]

for all \(\tilde{\Omega}(t_1, \infty)\). Hence, \(\Omega^*(t_1, \infty) \geq \tilde{\Omega}(t_1, \infty)\). From this and (A4),
\[(A6) \quad (r^{-1})\phi(Q^*(t_1))Q^*(t_1)e^{-rt} + \int_{t_1}^{\infty} (\phi(Q^*(t))Q^*(t) - \gamma(Q^*(t)))e^{-rt} dt \]

\[\geq (r^{-1})\phi(\tilde{Q}(t_1))\tilde{Q}(t_1)e^{-rt} + \int_{t_1}^{\infty} (\phi(\tilde{Q}(t))\tilde{Q}(t) - \gamma(\tilde{Q}(t)))e^{-rt} dt \]

But since \(\tilde{Q}(t_1) = Q^*(t_1)\) by assumption, the first terms on each side of (A6) are equal. The result follows immediately. Q.E.D.

**Proposition 3:** For any given plan \((q_t, \ t=0,1,\ldots)\), either of these best-price provisions yields the same expected discounted profits as simple price provisions. In other words, \(E_0\Omega = E_0\Gamma = E_0\Pi\).

**Proof:** From equations (7), (10), and (13), it follows immediately that

\[(A7) \quad P_{t'} = \rho_{t'} - E_{t'} \sum_{t'+1}^{\infty} \delta^{(t-t')}(\pi_{t-1} \cdot \rho_t) - \alpha_{t'} - \delta E_{t'}(\alpha_{t'} - P_{t'+1}) \]

The results below follow from algebraic manipulation of equations (9), (12), and (15), and substitution from equation (A7).

\[E_0\Omega = (\rho_0q_0 - \gamma_0(q_0)) + E_0 \sum_{t=1}^{\infty} \delta^t(\rho_tq_t - (\rho_{t-1}\cdot \rho_t)Q_{t-1} - \gamma_t(q_t)) \]

\[= (\rho_0q_0 - \gamma_0(q_0)) + E_0 \sum_{t=1}^{\infty} \delta^t(\rho_tq_t - \gamma_t(q_t)) - E_0 \sum_{t=1}^{\infty} \sum_{s=1}^{t-1} \delta^t(\rho_{t-1}\cdot \rho_t)q_s \]

\[= (\rho_0q_0 - E_0 \sum_{t=1}^{\infty} \delta^t(\rho_{t-1}\cdot \rho_t)q_0 - \gamma_0(q_0)) \]

\[+ E_0 \sum_{t=1}^{\infty} \delta^t(\rho_tq_t - E_t) \sum_{s=t+1}^{\infty} \delta^s(\rho_{t-1}\cdot \rho_t)q_s - \gamma_t(q_t)) \]
\[-(P_0 q_0 - \gamma_0(q_0)) + E_0 \sum_{t=1}^{\infty} \delta^t (P_q t - \gamma_t(q_t)) = E_0^\Pi \]

\[E_0^\Pi = (\alpha_0 q_0 - \gamma_0(q_0)) + E_0 \sum_{t=1}^{\infty} \delta^t (\alpha_t q_t - (\alpha_{t-1} - P_t) q_{t-1} - \gamma_t(q_t)) \]

\[-(\alpha_0 q_0 - \gamma_0(q_0)) + E_0 \sum_{t=1}^{\infty} \delta^t (\alpha_t q_t - \gamma_t(q_t)) - E_0 \sum_{t=0}^{\infty} \delta^{t+1} (\alpha_t P_{t+1} q_t) \]

\[-(\alpha_0 q_0 - \delta(\alpha_0 - E_0 P_1) q_0 - \gamma_0(q_0)) \]

\[+ E_0 \sum_{t=1}^{\infty} \delta^t (\alpha_t q_t - \delta E_t (\alpha_t P_{t+1} q_t) - \gamma_t(q_t)) \]

\[-(P_0 q_0 - \gamma_0(q_0)) + E_0 \sum_{t=1}^{\infty} \delta^t (P_q t - \gamma_t(q_t)) = E_0^\Pi \]

**Proposition 4**: By offering either of the best-price provisions defined above, the monopolist's choice of \(q_t^*\), \(t=0,1,...\) becomes time consistent.

**Proof**: Define \(\Omega(a,b)\) as follows:

\[(A8) \quad \Omega(a,b) = (\rho_a - \gamma_a(q_a)) + \sum_{t=a+1}^{b} \delta^t (\rho_t q_t - (\rho_{t-1} - \rho_t) Q_{t-1} - \gamma_t(q_t)) \]

Substitution of equation (11) into (A8) reveals that

\[(A9) \quad \Omega(a,b) = ((1-\delta)^{-1} \phi_a q_a - \gamma_a(q_a)) \]

\[+ \sum_{t=a+1}^{b} \delta^t ((1-\delta)^{-1} [\phi_t q_t - (\phi_{t-1} - \phi_t) Q_{t-1}] - \gamma_t(q_t)) \]

Note that \(\Omega(a,b)\) is independent of any actions taken either before time \(a\) or
after time $b$. Let $\Omega^*(a,b)$ refer to $\Omega(a,b)$ when the monopolist follows $(q_t^*, t=0,1,\ldots)$ from time $a$ through time $b$, and let $\bar{\Omega}(a,b)$ refer to $\Omega(a,b)$ when the monopolist follows $(\bar{q}_t, t=0,1,\ldots)$ instead. By definition,

$$\Omega^*(0,\infty) = \Omega^*(0,t_1-1) + \Omega^*(t_1,\infty) \geq \Omega^*(0,t_1-1) + \bar{\Omega}(t_1,\infty)$$

for all $\bar{\Omega}(t_1,\infty)$. Hence, $\Omega^*(t_1,\infty) \geq \bar{\Omega}(t_1,\infty)$.

The proof for one-period three-party best-price provisions is analogous.
REFERENCES


Holt, Charles A., and David T. Scheffman, "The Effects of Advance Notice and Best-Price Policies: Theory and Applications to Ethyl," mimeograph,
1985.


ENDNOTES

1 For a survey of this literature, see Ausubel and Deneckere (1986).

2 It may be difficult to locate a monopolist conforming exactly to Coase's example (constant marginal cost, infinite durability, etc.). Yet the type of behavior described in this literature is apparent in a variety of settings. For example, auto manufacturers have experienced chronic difficulties selling cars at "regular" prices. Consumers have come to expect such periodic incentives as rebates and discount financing, and have refused to purchase when these incentives are not offered.

3 The use of best-price provisions to resolve the durable good monopoly problem has been suggested, though not pursued, by Cooper (1984).

4 The similarity between best-price provisions and repurchase agreements is also discussed by Fng (1987).

5 With this discrete-time framework, the monopolist can commit to a level of output for the length of the period. Hence, there is a reduction in the time inconsistency problem facing the monopolist. See Stokey (1981) for details. The qualitative nature of the results does not change.

6 Cost uncertainties could also be introduced without changing the results.

7 If \( \rho_{t+k} > \rho_t \), then the provision requires an additional payment by the buyer. With the introduction of uncertainty, therefore, the term "best-price" provision is a misnomer except in those cases where prices can never
rise. Yet best-price provisions in practice rarely require buyers to make such payments, perhaps because the expected value of such payments to the monopolist is small relative to the costs of collecting them. In any event, the terminology will be retained.

8 Up to 1000 prints can be sold at time t=0 for a price \( p_0 = 5 + (0.953)E_0p_1 \).

9 In field market contracts for natural gas, pipelines (as monopsonistic buyers) agree to proration the quantities purchased from each buyer.