Econometric Analysis of Aggregation in the Context of Linear Prediction Models*

by

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Abstract

This paper deals with the problem of aggregation where the focus of the analysis is whether to predict aggregate variables using macro or micro equations. It generalizes the Grunfeld-Griliches prediction criterion to allow for contemporaneous covariances between the disturbances of micro equations, and the possibility of different parameteric restrictions on the equations of the disaggregate model. The paper also develops a formal statistical test of the hypothesis of 'perfect aggregation' which tests the validity of aggregation either through coefficient equality or through the stability of the composition of the regressors across the micro-units over time. The choice criterion and the perfect aggregation test are then applied to employment demand functions for the UK economy disaggregated by 40 industries.
1. Introduction

The problem of aggregation over micro units has been approached in the empirical literature from a number of different view points. In the case of linear models one important issue addressed in this literature is the problem of 'aggregation bias', defined by the deviation of the macro-parameters from the average of the corresponding micro-parameters. [See, for example, Theil (1954), Boot and de Wit (1960), Orcutt, Watts and Edwards (1968), Barker (1970), Gupta (1971), Sasaki (1978), and Winters (1980)].1 Another closely related issue is the prediction problem originally discussed by Grunfeld and Griliches (1960), where the focus of the analysis is whether to predict aggregate variables using macro or micro equations. Our primary concern in this paper is with the prediction problem in the context of linear models. We present a generalisation of the Grunfeld–Griliches (GG) prediction criterion which allows for contemporaneous covariances between the disturbances of the micro equations, and the possibility of different linear parametric restrictions on the equations of the disaggregate model. We also develop a formal statistical test of the hypothesis of 'perfect aggregation' which, unlike the test proposed by Zellner (1962) in the context of the seemingly unrelated regression model, does not necessitate the requirement that all coefficients across the equations of the disaggregated model are the same. The proposed test allows for the possibility of valid aggregation either through coefficient equality or through the invariance of the composition of the

(1) On the problem of aggregation across non-linear micro equations see, for example, Ando (1971), Kelejian (1980), Stoker (1984, 1986) and the references cited therein.
regressors across the micro-units over time.

The choice criterion and the test of perfect aggregation developed in the paper are then applied to two alternative specifications of employment functions for the UK economy disaggregated by 40 industries, and for the manufacturing sector disaggregated by 23 industries. As far as the choice criterion is concerned, the empirical results show that for the economy as a whole the disaggregate model fits better than the aggregate specification, while the reverse is true for the manufacturing industries taken as a group. The slightly better fit obtained for the aggregate model in the case of the manufacturing industries should not, however, be taken to mean that there are no aggregation problems at this level. In fact the application of the test of perfect aggregation to the employment functions provides strong evidence in favour of rejecting the hypothesis of perfect aggregation both for the economy as a whole, and for the manufacturing sector. Our results also suggest serious upward bias in the estimates of output and real wage elasticities of aggregate employment demand obtained for the UK in the literature using aggregate relations. The slightly better within-sample performance of the aggregate specification in the case of the manufacturing industries is best interpreted as an indication of the misspecification of the disaggregate equations.

The plan of the paper is as follows. Section 2 sets out the basic econometric framework. Section 3 examines the small sample bias of the GG prediction criterion. Section 4 generalises the basic model so that different specifications for the micro-equations is possible, and derives a goodness-of-fit criterion for discrimination between aggregate and disaggregate models that does not suffer from the
small sample problem. Section 5 considers alternative methods of testing for the errors of aggregation, and develops a new test of the hypothesis of perfect aggregation. Section 6 deals with the problem of misspecification of the disaggregate model and the implications that this has for the use of the proposed choice criterion. Section 7 contains a detailed application of the econometric methods developed in the paper to the UK employment functions.

2. The basic econometric framework

We start with the micro-model analysed by Theil (1954), and subsequently by Grunfeld and Griliches (1960), and others, and suppose that the \( n \) observations of the \( m \) micro-units \( (y_{it}, i = 1, 2, ..., m; t = 1, 2, ..., n) \) are generated according to the following linear specifications

\[
y_{it} = \sum_{j=1}^{k} \beta_{ij} x_{i,t} + u_{it}, \quad i = 1, 2, ..., m, \quad t = 1, 2, ..., n
\]

or in matrix notations (Kloek, 1961)

\[
(2.1) \quad \begin{pmatrix} y \end{pmatrix} = \begin{pmatrix} X \end{pmatrix} \begin{pmatrix} \beta \end{pmatrix} + \begin{pmatrix} u \end{pmatrix}
\]

In the above specification it is assumed that the variations in dependent variables of all micro-units can be explained by means of linear combination of the same set of \( k \) explanatory variables. This assumption will be relaxed in the next section.

Writing (2.1) as a System of Seemingly Unrelated Equations (SURE), following Zellner (1962) we have
(2.2) \[ \mathbf{y} = X \mathbf{\beta} + \mathbf{u} \]

where \( \mathbf{y} = (y_1, y_2, \ldots, y_m)' \), \( \mathbf{\beta} = (\beta_1', \beta_2', \ldots, \beta_m')' \), \( \mathbf{u} = (u_1', u_2', \ldots, u_m')' \), and \( X \) is an \( mn \times mk \) block-diagonal matrix of full column rank with matrix \( X_i \) as its \( i \)th block. We also make the following assumption:

**Assumption 1**: The \( mn \times 1 \) disturbance vector \( \mathbf{u} \) is distributed independently of \( X \), has mean zero and the variance matrix \( \mathbf{\Sigma} = \mathbf{\Sigma} \otimes \mathbf{I}_n \), where \( \mathbf{\Sigma} = (\sigma_{ij}) \), and \( \mathbf{I}_n \) is the identity matrix of order \( n \).

The problem of aggregation can arise when an investigator interested in the behaviour of the macro-variable \( y_a = \sum_{i=1}^{m} y_i ' \) considers the single macro-equation

(2.3) \[ H_a: y_a = X_a b + \mathbf{v}_a \]

where \( X_a = \sum_{i=1}^{m} X_i \), instead of the \( m \) micro-equations in (2.1).

Following Grunfeld and Griliches (1960) we examine the question of whether to predict \( y_a \) using the macro-equation (2.4), or the micro-equation (2.3), or the micro-equations (2.1).

3. **The small sample bias of the Grunfeld-Griliches criterion**

The GG prediction (or more accurately the within-sample goodness-of-fit) criterion for the discrimination between the disaggregate model, \( H_d \) and the aggregate model, \( H_a \) can be written as
Choose $H_d$ if $e_d e_d < e_a e_a$, otherwise choose $H_a$.

where $e_d$ and $e_a$ are the estimates of the errors in predicting $y_a$ under $H_d$ and $H_a$ respectively. The estimates employed by GG for $e_d$ and $e_a$ are based on the Ordinary Least Squares (OLS) method and are given by

\[(3.1) \quad e_a = M_a y_a, \quad M_a = I_n - X_a (X_a' X_a)^{-1} X_a' = I_n - A_a,\]

and

\[(3.2) \quad e_d = \sum_{i=1}^{m} M_1 y_i, \quad M_1 = I_n - X_1 (X_1' X_1)^{-1} X_1' = I_n - A_i.\]

It is important to note that in general $e_d$ is not an efficient estimator of $y_d = y_a - \sum_{i=1}^{m} X_i \beta_i$, unless the disturbances of the micro-equations are contemporaneously uncorrelated (i.e. $\sigma_{ij} = 0$, for $i \neq j$), or when $X_i$ can be written as exact linear functions of $X_a$.

The problem of efficient estimation of $\beta_i$, and hence $y_d$, and the effect that this has for the GG criterion will be discussed later. For the moment we assume that the GG criterion, as specified above, is applied even in the case where the micro-equation disturbances are contemporaneously correlated, and investigate the small sample bias that such a procedure entails.

Like the justification offered for Theil's $R^2$ criterion, the rationale behind the use of the GG criterion must lie in the fact that if the micro-equations are correctly specified, then 'on average' the fit of $y_a$ from the macro-equation should not be any better than that obtained from the micro-equations. That is we should have
(3.3) \[ E_d(e'_d e_d) \leq E_d(e'_a e_a) \]

where \( E_d(\cdot) \) represents the mathematical expectations operator under 
\( H_d \). However, using (3.1) and (3.2) it is easily seen that(1)

\[ E_d(e'_d e_d) - E_d(e'_a e_a) = -E(\xi'^t M_a \xi) - 2 \sum_{s=1}^{k} \sum_{i>j}^{m} \sigma_{ij} (1 - E(\phi_{s,ij}^2)) \]

where \( \xi = \sum_{i=1}^{m} x_i \beta_i - x_a b \), and \( \phi_{s,ij} \) is the sth canonical correlation coefficient between the explanatory variables of the ith and the jth micro-equations. Therefore, in general the inequality condition (3.3) need not be satisfied even if \( H_d \) is correctly specified. There are, however, two circumstances under which the the GG criterion satisfies the inequality relationship (3.3).

(i) when \( X_i \) can be written as exact linear functions of \( X_j \), for all \( i \) and \( j \). In this case \( \phi_{s,ij}^2 = 1 \), and irrespective of the values of \( \sigma_{ij} \) we have \( E_d(e'_d e_d) - E_d(e'_a e_a) = -E(\xi'^t M_a \xi) \).

(ii) when the micro-disturbances are all contemporaneously uncorrelated \( (\sigma_{ij} = 0, i \neq j) \). In general the direction of the bias involved in the use of the GG criterion in small samples depends on the signs of \( \sigma_{ij} \), for \( i \neq j \).

The finite sample bias in the use of the GG criterion will not disappear even when \( \beta_i \) are estimated efficiently by the SURE method.

Consider the simple case where \( \Sigma \) is known. The SURE estimator of \( H_d \),

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(1) In deriving this result we have also made use of the relation

\[ k - \text{Tr}(A_i A_j) = \sum_{s=1}^{k} (1 - \phi_{s,ij}^2) > 0 \].

See, for example, Rao (1973, pp. 582-587).
which we denote by \( e_S \), will be

\[
\varepsilon_S = S(I_{nm} - A)y
\]

where \( S \) stands for the \( n \times nm \) summation matrix

(3.4) \( S = [I_n; I_n; \ldots ; I_n] \),

and

(3.5) \( A = X(X'\Omega^{-1}X)^{-1}X'\Omega^{-1} \).

Under \( H_d \), \( e_S = S(I_{nm} - A)y \), and hence

\[
E_d(e'_S e_S) - E_d(e'_a e_a) = k \sigma^2_a - E(\xi' M_a \xi) - E(\text{Tr}[(X'\Omega^{-1}X)^{-1}X'SS'X])
\]

where \( \sigma^2_a = \sum_{i,j=1}^{m} \sigma_{ij} \). Again leaving the case where \( X_a \) are exact linear functions of \( X_d \) to one side, the strict inequality \( E_d(e'_S e_S) \neq E_d(e'_a e_a) \) holds only in the special case where \( \sigma_{ij} = 0 \), for \( i \neq j \).

4. A generalised goodness-of-fit criterion for discrimination between aggregate and disaggregate models

From the results of the previous section it is now a straightforward matter to derive a choice criterion for discrimination between the disaggregate and the aggregate models that does not suffer from the finite sample bias of the GG criterion. But it is first important to extend the econometric framework of section 2, so that different specifications for the micro-equations can be considered. Such a generalisation is particularly important when the primary purpose of the disaggregation is to achieve a better explanation of the macro-
variables. Accordingly, we consider the following specifications for the disaggregate and the aggregate models:

\[ \tilde{H}_d : y_{i1} = x_{i1} \beta_1 + u_{i1} \quad i = 1, 2, \ldots, m , \]
\[ \tilde{H}_a : y_a = \begin{pmatrix} x_a \beta \end{pmatrix} + v_a , \]

where \( \text{Rank}(X_i) = k_i \), and \( \text{Rank}(X_a) = k_a \). In this formulation there are no restrictions on the number of columns of \( X_i \), or what these columns may represent. The micro-equations under \( \tilde{H}_d \) can also be viewed as a restricted version of the equations under \( H_d \), with each micro-equation having its own specific linear parametric restrictions. In this way a wide range of different specifications across the micro-equations can be allowed for. The specification of the macro-equation is also generalized so that the investigator can specify a restricted form of the macro-equation defined in (2.3).

Consider now the following 'adjusted' goodness-of-fit criteria for the aggregate and the disaggregate models

\[ s_a^2 = \frac{e_{a}^{'}e_{a}}{(n - k_a)} , \]

and

\[ s_d^2 = \frac{1}{m} \sum_{i,j=1}^{m} \hat{\sigma}_{ij} , \]

where

\[ \hat{\sigma}_{ij} = \left( n - k_i - k_j + \text{Tr}(A_i A_j) \right)^{-1} e_{i}^{'}e_{j} , \]
with \( \varepsilon_a \) and \( \varepsilon_i \) being respectively the OLS residual vectors of the regressions under \( \tilde{H}_a \) and \( \tilde{H}_d \), and \( A_i = X_i(X'_iX_i)^{-1}X_i' \). The use of \( s_d^2 \) as a measure of the goodness-of-fit of the disaggregate model is justified on the grounds that it represents an unbiased (and consistent) estimator of \( \sigma_a^2 = V( \sum_{i=1}^{m} u_{it} ) \), the population variance of the error of predicting \( y_a \) from the disaggregate model. It is now easily seen that under \( \tilde{H}_d \),

\[
(4.4) \quad E_d(s_d^2) - E_d(s_a^2) = - (n - k_a)^{-1} E(\xi'_a \mathbf{M}_a \xi) < 0,
\]

where \( \xi \) is now defined by

\[
(4.5) \quad \xi = \sum_{i=1}^{m} X_i \beta_i - X_a \beta .
\]

Therefore, as required we have \( E_d(s_d^2) < E_d(s_a^2) \), and unlike the GG criterion, the use of the proposed goodness-of-fit criteria \( s_a^2 \) and \( s_d^2 \) will 'on average' result in the choice of the disaggregate model in finite samples, assuming, of course, that the disaggregate model is correctly specified. In situations where the disaggregate model fits worse than the aggregate model (i.e. \( s_d^2 > s_a^2 \)), it is likely that the disaggregate model is misspecified. The implications for the above choice criterion when the disaggregate model is subject to errors of specification will be discussed below. Here, for comparison purposes it is worth considering the following decomposition of the \( s_d^2 \) criterion.
\[ (4.6) \quad s_{d}^{2} = (n - k_{a})^{-1} e_{d} e_{d} + (n - k_{a})^{-1} \sum_{i=1}^{m} (k_{i} - k_{a}) \hat{\sigma}_{i} \]

\[ + 2(n - k_{a})^{-1} \sum_{i>j}^{m} \phi_{ij} \left( 1 - \phi_{ij} \right) e_{i} e_{j}, \]

where \( e_{d} = \sum_{i=1}^{m} e_{i} \), and

\[ \phi_{ij} = (n - k_{a})^{-1} (k_{i} + k_{j} - k_{a} - \text{Tr}(A_{i}A_{j})) . \]

The GG prediction criterion focuses on the first term on the right-hand side of (4.6) and ignores the asymptotically negligible second and third terms. The second term represents the contribution to the \( s_{d}^{2} \)-criterion arising out of the possible differences in the number of estimated coefficients between the aggregate and the disaggregate models. The third term in (4.6) captures the effect of the contemporaneous correlation amongst the disturbances of the micro-equations.

5. Tests of aggregation

In studying the aggregation problem our emphasis so far has been on the model selection procedures. An alternative approach would be to employ classical hypothesis testing procedures and develop a statistical test of the conditions necessary for valid aggregation. In the context of the generalized disaggregate model \( \tilde{M}_{d} \), the necessary condition for perfect aggregation is given by \( \xi = 0 \), where \( \xi \) is defined in (4.5). Under the hypothesis of 'perfect aggregation'
\[ H_\zeta : \xi = \sum_{i=1}^{m} x_i \beta_1 - x_0 \beta = 0, \]

it readily follows from (4.4) that \( E_d(s_d) = E_d(s_a^2) = \sigma_a^2 \), and as far as the fit of \( y_a \) is concerned we should not expect to gain from disaggregation.\(^{(1)}\)

Before developing a formal test of \( H_\zeta \), it is important to note that the condition \( \xi = 0 \) can be given a meaningful interpretation only in the context of the basic model (2.1) where \( \beta_i \) are of the same dimension and refer to the same type of variables across the micro-equations. In this case the condition \( \xi = 0 \) is clearly satisfied under the 'micro-homogeneity' hypothesis,\(^{(2)}\)

\[ H_\beta : \beta_1 = \beta_2 = \ldots = \beta_m. \]

This is not, however, the only situation where \( H_\zeta \) holds. Another hypothesis of interest which yields \( \xi = 0 \), is the 'compositional stability' hypothesis

\[ H_\times : x_i = x_a c_i, \quad i = 1, 2, \ldots, m, \]

where \( c_i \) are \( k \times k \) non-singular matrices of fixed constants, such that \( \sum_{i=1}^{m} c_i = I_k \). The 'compositional stability' hypothesis represents a set of restrictions on the joint probability distribution of the regressors and states that the composition of the regressors across

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\(^{(1)}\) For the basic disaggregated model (2.1), the hypothesis \( H_\zeta \) is equivalent to the \( n \)-covariance condition discussed in Theil (1954) and Lancaster (1966), in the special case where the number of regressors is equal to one.

\(^{(2)}\) Notice that this hypothesis can not hold under the generalized disaggregated model \( H_d \).
micro-units remain fixed over time. This condition for valid aggregation in linear models has been discussed in the econometric literature by Klein (1953) and Wold and Juréen (1953). Distributional assumptions on the regressors have also been employed in the literature, for example, by Ando (1971), McFadden and Reid (1975), Kelejian (1980), and more recently by Stoker (1984) to connect the aggregate function to the underlying micro-equations in the context of non-linear models. Under $H_X$, the macro-coefficient vector $\beta$, is defined in terms of the micro-coefficients through the identity

$$ b = \sum_{i=1}^{m} C_i \tilde{b}_i. $$

The condition $G = 0$ will also be met under the mixed hypothesis (1)

$$ H_{G_X} : \begin{cases} \beta_{s+1} = \beta_{s+2} = \ldots = \beta_m = \beta_1 \end{cases} $$

where in this case

$$ X = \sum_{i=1}^{s} X_i, \quad \sum_{i=1}^{s} C_i = I_k \text{ and } \tilde{b}_i = \sum_{i=1}^{s} C_i \beta_i. $$

The test proposed by Zellner (1962) for aggregation bias is a test of the micro-homogeneity hypothesis, $H_B$, and is not necessarily relevant as a test of $H_G$: $G = 0$. The Zellner test can therefore be unduly restrictive. Rejection of $H_B$ does not necessarily imply that the perfect aggregation hypothesis $H_G$ should also be rejected. What is needed is a direct test of $G = 0$. In what follows we develop such a test in the case of the basic disaggregated model (2.1) and the

(1) The aggregation condition is also met by an alternative mixed hypothesis where the $k$ regressors $X_i$ can be partitioned into two sub-sets, one of which satisfies the compositional stability hypothesis and the other having an associated parameter vector satisfying the micro-homogeneity hypothesis.
aggregate model (2.3). Although our results can be extended to the
generalized model $\tilde{H}_d$, we have chosen not to do this here, since we do
not think that the perfect aggregation condition $\xi = 0$ can be given a
plausible interpretation under $\tilde{H}_d$. In the case of the generalized
model neither the micro-homogeneity hypothesis nor the compositional
stability hypothesis can be maintained.

5.1 A test of perfect aggregation: case of known $\Sigma$

To help clarify the nature of the test that we are proposing, we
first develop the test in the case where $\Sigma$, the covariance matrix of
the micro-disturbances, is known. A computationally feasible version
of the test will be discussed in Section 5.2.

The idea behind the test is straightforward and asks whether the
estimator of $\xi$ is significantly different from zero. When $\Sigma$ is known
an efficient estimator of $\xi$ is given by

$$\tilde{\xi} = S \tilde{\beta} - \tilde{x}_a \hat{\beta},$$

where $\tilde{\beta}$ and $\hat{\beta}$ are the SURE and the OLS estimators of the parameters
of the disaggregate and the aggregate equations respectively, and $S$
is the summation matrix defined by (3.4). Substituting

$$\tilde{\beta} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y, \quad \hat{\beta} = (X'X_a)^{-1}X'Y_a$$
in (5.1) now yields $\tilde{\xi} = Hy$

where $H = SA - A_a S$. The matrices $A_a$ and $A$ are already defined by

(3.1) and (3.5), respectively. On the null hypothesis that

$$\xi = \Sigma \sum_{i=1}^{m} x_i \beta_i - x_a \hat{\beta} = 0,$$

we have $\tilde{\xi} = Hy$. Therefore, under the

assumption that $y$ is normally distributed with zero means and a known
non-singular variance matrix $\Omega = \Sigma \otimes I_n$.
$$\hat{\xi} = (\hat{H} \hat{O} \hat{H}')^{-1} \hat{\xi} \sim \chi^2_n.$$  

A necessary condition for $H \hat{O} \hat{H}'$ to have a full rank can be obtained in the following manner: since, by assumption $\Omega$ is a non-singular matrix then $\text{Rank}(H \hat{O} \hat{H}') = \text{Rank}(H)$. But,

$$\text{Rank} (H) \leq \text{Rank} (SA) + \text{Rank} (A_a S),$$

$$\text{Rank} (A_a S) = \text{Rank} (A_a) = k,$$

$$\text{Rank} (A) = \text{Tr}(A) = mk, \quad \text{Rank} (S) = n$$

and

$$\text{Rank} (SA) \leq \text{Min}(n, mk).$$

Consequently, $\text{Rank} (H) \leq k + \text{Min}(n, mk)$, and for matrix $H$ to have full rank equal to $n$, it is necessary that $k + \text{Min}(n, mk) \geq n$, or

$$k(m+1) \geq n.$$  

This rank condition is clearly satisfied when $m$ is large relative to $n/k$. But in situations where the number of micro-equations is relatively large, the computational burden of obtaining the SURE estimates, $\hat{\beta}$ in (5.1) can be considerable. One possibility would be to construct a test of $H_0$ based on the OLS estimates of $\beta$ instead of the SURE estimates. The estimate of $\xi$ based on the OLS estimators is given by

$$\hat{\xi} = \sum_{i=1}^{m} x_i \hat{\beta}_i - X \hat{\beta}_a = e_a - e_d,$$

where $e_a$ and $e_d$ are already defined by (3.1) and (3.2), respectively.
Under $H_d$, and on the assumption that the hypothesis of perfect aggregation $H_C$ holds, we have

$$
\hat{\xi} = \sum_{i=1}^{m} (A_i - A_a) u_i = \sum_{i=1}^{m} H_i u_i .
$$

Now assuming that $u_i$ are normally distributed, then conditional on $X_i$, we have

$$m^{-1/2} \hat{\xi} | X_i \sim N(0, \Psi_m) ,$$

where

$$
\psi_m = m^{-1} \sum_{i,j=1}^{m} \sigma_{ij} H_i H_j .
$$

Therefore, assuming that $\psi_m$ is a non-singular matrix, we arrive at the result

$$m^{-1} (e_a - e_d) \psi_m^{-1} (e_a - e_d) \sim \chi^2_n ,$$

which is the OLS counterpart of (5.2).

5.2 Case of unknown $\Sigma$

When $\Sigma = (\sigma_{ij})$ is unknown, it is still possible to obtain an 'approximate' test of the perfect aggregation hypothesis by replacing $\sigma_{ij}$ in (5.2) or (5.6) with their SURE or the OLS estimates. Here, we focus on the latter and consider testing $H_g$ by means of the statistic

$$a_m = m^{-1} (e_a - e_d) \psi_m^{-1} (e_a - e_d) ,$$

(1) Notice that a necessary condition for $\psi_m$ to be invertible is given by (5.3).
where

\begin{equation}
\hat{\psi}_m = m^{-1} \sum_{i,j=1}^{m} \hat{\sigma}_{ij} H_i H_j,
\end{equation}

\begin{equation}
\hat{\sigma}_{ij} = (n - 2k + \text{Tr}(A_{i,j} A_{j,i}))^{-1} e_i e_j.
\end{equation}

We shall refer to a test of \( H_C \) based on (5.7) as the perfect aggregation test, or the \( a \)-test for short.

The exact distribution of the \( a_m \) statistic under \( H_C \) is no longer a \( \chi^2_n \), and unfortunately does not lend itself to a simple derivation either. But it is possible to approximate the distribution of \( a_m \) by means of a 'suitable' limiting distribution. The usual asymptotic theory where the limiting distribution is obtained by letting \( n \), the sample size, tend to infinity is clearly not applicable here. A relevant asymptotic framework for testing the hypothesis of perfect aggregation is to allow the level of disaggregation, \( m \), to increase without a bound, while keeping the sample size \( n \), fixed. The idea of expanding the micro units to obtain distribution properties in an aggregate framework is not particularly novel and has been used by Powell and Stoker (1985), and Granger (1987). In our application of the large \( m \)-asymptotics we make the following assumptions:

**Assumption 2:** The average matrix \( \bar{X}_m = m^{-1} X_m \), and the aggregate projection matrix \( A_m = \bar{X}_m (\bar{X}_m^\prime \bar{X}_m)^{-1} \bar{X}_m^\prime \), converge (in probability) to finite limits.

**Assumption 3:** The elements of the disaggregate projection matrices, \( A_i = X_i (X_i^\prime X_i)^{-1} X_i^\prime \), remain bounded in absolute value as \( m \to \infty \).

Notationally we write \(|A_i| < P < \infty\).
Assumption 4: The elements of the variance matrix $\Sigma = (\sigma_{ij})$ remain bounded as $m \to \infty$. Namely, $|\sigma_{ij}| < \tau^2 < \infty$, $\forall i,j$.

Assumption 5: the variance matrix $\psi$ defined by

$$\psi_m = m^{-1} \sum_{i,j=1}^{m} \sigma_{ij} H_i H_j$$

tends to a non-singular matrix $\psi$, as $m \to \infty$.

When assumptions 1-5 hold, it seems reasonable to suppose that the distribution of $a_m$, on the null hypothesis of perfect aggregation will tend towards a $\chi^2_n$ as $m \to \infty$. Although, at this stage we are not able to present a proof of this proposition in its present form, we can nevertheless offer the following less general theorem.

**Theorem:** consider the disaggregate model (2.1) and suppose that the standardised micro-disturbances $u_{it}/\sqrt{\sigma_{ii}}$ are identically distributed, independently both across time periods and across equations, with zero means, unit variances and finite third order moments. Then conditional on $X$, and under assumptions 2-5, the statistic

$$a_m = (e_a - e_d)' \left( \sum_{i=1}^{m} \hat{\sigma}_{ii} H_i H_i' \right)^{-1} (e_a - e_d),$$

will be asymptotically distributed as a $\chi^2_n$ variate on the null hypothesis of perfect aggregation (i.e. $\xi = 0$), as $m \to \infty$.

(See Appendix A for a proof)
It is worth noting that the above theorem is applicable even when micro-equations contain lagged dependent variables, macro-variables, or other common variables such as an intercept term or a time trend.\(^1\)

6. **Disaggregation and specification error**

The model selection criterion and the aggregation test developed in this paper are based on the assumption that the disaggregate model is correctly specified. In reality, however, both the disaggregate and the aggregate models may suffer from errors of specification, with the latter also being subject to the additional problem of aggregation error. In such a circumstance the issue of whether disaggregation is useful for the study of macro-phenomena and the extent of the gain that may be expected from disaggregation depends very much on the relative importance of the two types of errors of specification and aggregation. In this section the implications that errors of specification may have for the use of our proposed choice criterion will be examined.

Let the correctly specified disaggregate model be

\[
y_{i t} = X_i \beta_1 + W_t \gamma_1 + u_{i t}, \quad i = 1, 2, \ldots, m
\]

\[
\begin{array}{c}
n \times 1 \\
\times k_t \times 1 \\
\times s_t \times 1 \\
\times 1
\end{array}
\]

which in a stacked form can also be written as

---

1. Here it is assumed that the micro-disturbances, \(u_{i t}\), are serially uncorrelated; otherwise the estimates can be badly biased if lagged values of the dependent variable are included in the disaggregate equations.
(6.2) \[ y = \bar{x} \bar{s} + W \bar{y} + \bar{y}, \]

where \( X \) is now an \( mn \times k \) (where \( k = \sum_{i=1}^{m} k_i \)) block diagonal matrix with \( X_i \) as its \( i \)th block, \( \bar{x} = (\chi_1', \chi_2', \ldots, \chi_m') \), and \( W \) is an \( mn \times \bar{s} \), (where \( \bar{s} = \sum_{i=1}^{m} s_i \)) block-diagonal matrix with \( W_i \) on its \( i \)th block. The other notations are as in relation (2.2). Suppose now that a researcher misspecifies this model by omitting the variables in \( W \), and continues to employ the model selection criterion based on \( s_a^2 \) and \( s_d^2 \), defined by (4.1) and (4.2) respectively. Clearly, the result \( E_d(s_d^2) \leq E_d(s_a^2) \), which provided the rationale for the choice criterion, need no longer hold.

Stacking the OLS residuals \( \bar{e}_i = M_i y_i \) in the vector \( \bar{e} = (\bar{e}_1', \bar{e}_2', \ldots, \bar{e}_m') \), \( s_d^2 \) can also be written as \( s_d^2 = \bar{e} \bar{L} \bar{e} \), in which \( \bar{L} = (\Lambda \otimes I_n) \), and \( \Lambda \) is an \( m \times m \) matrix with a typical element equal to \( [\text{Tr}(M_i M_j)]^{-1} \). Now under the correctly specified model (6.2),

\[
\bar{e} = M y ,
\]

\[
M = I_{mn} - X (X'X)^{-1} X' ,
\]

\[
= MW + Mu .
\]

Hence

(6.3) \[ E_d(s_d^2 | X, W) = \sigma_a^2 + \chi' \bar{W} M M \bar{W} \chi . \]

Since in general \( L \) may not be a positive semi-definite matrix, without further information about the nature of the specification error, it will not be possible to say whether misspecification leads to an upward or a downward bias in the application of the choice criterion. Expanding (6.3) in terms of the misspecification of the individual micro-equations we have
(6.3) \[ E_d(s^2_d | X, W) = \sigma_a^2 + (n - k_a)^{-1} \sum_{i=1}^{m} d_i^2 d_i + 2 \sum_{i>j}^{m} \frac{d_i^2 d_j}{\text{Tr}(M_i M_j)} \]

where \( d_i = M_i W_i y_i \), and \( \text{Tr}(M_i M_j) = n - k_i - k_j + \text{Tr}(A_i A_j) \).

The direction of the bias resulting from misspecification clearly depends on the sign of the cross-equation terms \( d_i^2 d_j \), \( i \neq j \), and their quantitative importance relative to the equation-specific terms \( d_i^2 d_i \). In practice, however, it is reasonable to expect that \( E_d(s^2_d) > \sigma_a^2 \).

Now turning to the \( s_a^2 \) criterion, under (6.1) we obtain

(6.4) \[ E_d(s^2_a | X, W) = \sigma_a^2 + (n - k_a)^{-1} \sum_{i=1}^{m} X_i^2 \]

where

(6.5) \[ \xi = \sum_{i=1}^{m} X_i y_i + \sum_{i=1}^{m} W_i y_i = \xi_a + \xi_s \]

Comparing (6.3) and (6.4) it is clear that in general it is not possible to say whether \( E_d(s^2_a) \) exceeds \( E_d(s^2_d) \). The result depends on the relative importance of the specification error and the aggregation error for the explanation of the macro-variable \( y_a \). In their work, Grunfeld and Griliches (1960), consider a special case of some interest where there are micro-specification errors that cancel out in the aggregate. In the context of model (6.1) this can arise either when there are, for example, errors of measurement in the micro-variables that cancel out exactly in the aggregate (1) (i.e.

---

(1) The problem of measurement errors in a disaggregate model in the special case where \( m = k = 2 \) is discussed by Aigner and Goldfeld (1974).
\[ \xi_s = \sum_{i=1}^{m} W_i x_i = 0 \], or when the micro-specification errors involve omission of macro-variables already included in the aggregate model, \(^{(1)}\) (i.e. \( M_a \xi_s = 0 \)). In such a case, using (6.4), we have

\[ E_d(s_a^2 | X, W) = \sigma_a^2 + (n - k_a)^{-1} \xi_a^T M_a \xi_a , \]

and only aggregation errors (\( \xi_a \neq 0 \)) cause the expectations of \( s_a^2 \) to exceed the true error variance of the aggregate model. However, even in this special case it is not possible to say whether it is better to use the aggregate model. The answer still depends on the relative importance of the micro-specification errors in the disaggregate model and the aggregation error in the aggregate model for the explanation and prediction of macro-behaviour. The issue of whether one should choose the aggregate or the disaggregate model cannot be resolved by a priori reasoning alone and has to be settled with respect to particular problems and in the context of specific models.

7. Applications: employment demand functions in the UK

In this section the methods described in the preceding sections will be applied to the annual estimates of disaggregate and aggregate

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\(^{(1)}\) It is beyond the scope of the present paper to go into the reasons for the importance of macro-variables in the explanation of micro-behaviour. In general they may arise because individual micro-behavioural relations are not independent but are influenced or constrained by outcomes (or expectations of outcome) of the market as a whole.
employment demand functions for the UK economy. Although, our emphasis will be on the aggregation problem, it is hoped that the disaggregate results are of some interest in their own right.

Our empirical analysis is based on the Cambridge Growth Project Databank and uses a consistent set of data on man hours \( (EH_i) \), outputs \( (Y_i) \), and real product wages \( (W_i) \) across 41 industry groups. Details of the data and the sources are given in the data appendix. For the employment equation at the industry level we have adopted the following fairly general log-linear dynamic specification

\[
(7.1) \quad LEH_{it} = \beta_{i1} \frac{1}{m} + \beta_{i2} \left( T_{t-1} / m \right) + \beta_{i3} LEH_{i, t-1} + \beta_{i4} LEH_{i, t-2} \\
+ \beta_{i5} LY_{it} + \beta_{i6} LY_{i, t-1} + \beta_{i7} LW_{it} + \beta_{i8} LW_{i, t-1} \\
+ \beta_{i9} \left( SLYT_{t-1} / m \right) + \beta_{i10} \left( SLYT_{t-1} / m \right) + u_{it},
\]

\[ i = 1, 2, 3, 5, 6, \ldots, 41, \]
\[ t = 1956, 1957, \ldots, 1984, \]

where

- \( LEH_{it} \) = log of man-hours employed in industry \( i \) at time \( t \),
- \( T_{t} \) = time trend \( (T_{1980} = 0) \),
- \( LY_{it} \) = log of industry \( i \) output at time \( t \),
- \( LW_{it} \) = log of average real wage rate per man-hours employed in industry \( i \) at time \( t \),
- \( SLYT_{t} \) = \( \sum_{i=1}^{41} {LY}_{it} \).

Industry 4 (Mineral oil and natural gas) is excluded from the analysis, on the grounds that output and employment in this industry
have been negligible before 1975.

The above specification for the employment demand function can be justified theoretically when employment decisions are made at the industry level by cost minimizing firms with identical production functions and the same given demand and factor price expectations. In this framework the inclusion of lagged employment variables can be justified on the grounds of inertia in revision of expectations, adjustment costs involved in hiring and firing of workers, or aggregation over different labour types. (See, for example, Sargent, 1978, and Nickell, 1984). The variable SLYT$_t$, which measures the level of aggregate output (in logs) is a proxy measure intended to capture changes in demand expectations arising from the perceived interdependence of the demand in economy by the firms in the industry. (1) The time trend is included in the specification in order to allow for the effect of neutral technical progress on the labour productivity. (2) Ideally, we would have liked to avoid using a simple time trend as a proxy for the trend productivity. But, unfortunately direct reliable observations on technical change, especially at the

(1) Apart from the aggregate variable SLYT$_t$, the employment function (7.1) is similar to the equations estimated by Peterson (1988), as a part of the Cambridge Multisectoral Dynamic Model of the UK economy. (See Barker and Peterson, 1988).

(2) Notice that, for the ease of comparison of the aggregate and the disaggregate parameter estimates the time trend, and the aggregate output variable that are common to all the micro-equations are specified in the 'average' form. Clearly this has no effect on the overall fit of the equations for a fixed level of disaggregation.
industry level are not available.\(^1\) The use of time trends in regression equations with non-stationary variables also poses a number of important econometric problems and, as shown by Nelson and Kang (1983), Mankiw and Shapiro (1985, 1986), and Durlauf and Phillips (1986), can result in biased inferences.\(^2\) In view of these measurement and econometric problems it is not clear how one should proceed to allow for trend changes in labour productivity on employment demand functions.\(^3\) Here, in the absence of direct measures of trend productivity at the industry level we estimate (7.1) with a time trend, but also briefly report on the effects of omitting the time trends.\(^4\)

For the aggregate employment function we adopted the following dynamic specification

\(^1\) In their work on aggregate employment demand function Layard and Nickell (1985, p. 168) use a production function approach to obtain an index of labour-augmenting technical progress as a 'residual'. This approach requires time series data on capital stock and the share of capital which are not readily available at the industry level. Moreover, since their measure of technical progress is constructed using actual employment, including it as a regressor in the employment demand function can lead to biased estimates.

\(^2\) Notice, however, that in the case of the test of perfect aggregation where the test is justified asymptotically for a fixed sample size but with an increasing number of micro-units, the inclusion of time trends in the micro-equations does not affect the validity of the test.

\(^3\) However, see Harvey et al. (1986) where a stochastic specification (a random walk with a drift) is advanced for trend productivity. In their formulation \(T_t = a + T_{t-1} + \epsilon_t\), where \(\epsilon_t\) is a white-noise process.

\(^4\) The effect of replacing the time trend by other proxies such as distributed lag functions of gross investment as a way of modelling endogenous technical change à la Kaldor (1956) is discussed in Lee et al. (1988).
(7.2) \[ \text{SLET}_t = b_1 + b_2 T_t + b_3 \text{SLET}_{t-1} + b_4 \text{SLET}_{t-2} + b_5 \text{SLYT}_t \]

\[ + b_6 \text{SLYT}_{t-1} + b_7 \text{SLWT}_t + b_8 \text{SLWT}_{t-1} + u_t, \]

\[ t = 1956, 1957, \ldots, 1984, \]

where

\[ \text{SLET}_t = \sum_{i=1}^{41} \text{LEH}_{it}, \quad \text{and} \quad \text{SLWT}_t = \sum_{i=1}^{41} \text{LW}_{it}. \]

\[ \text{SLET}_t = \sum_{i=1}^{41} \text{LEH}_{4it}. \quad \text{and} \quad \text{SLWT}_t = \sum_{i=4}^{41} \text{LW}_{4it}. \]

Here we are assuming that the purpose of the study is to explain SLET, which is the sum of the logarithms of industry employment (in man-hours). This is clearly different from the more usual practice of specifying aggregate employment functions in terms of the logarithm of the sum of industry employment. For our purposes the specification (7.2) has the advantage that it fits directly within the theoretical framework of the paper, and as is pointed out, for example, by Lovell (1973), it also satisfies the Klein-Nataf consistency conditions. A theoretical analysis of the alternative methods of aggregating micro-specifications such as (7.1), and an econometric investigation of the relative merits of such aggregation methods is beyond the scope of the present paper.

7.1 Results for the economy as a whole

The estimates of the unrestricted version of the industry demand functions (7.1) for the 40 industry groups 1, 2, 3, 5, 6, ..., 41, over the sample period 1956-84 are set out in Table 1. The estimates of the standard errors of the regression coefficients are given in the brackets. The Table also includes the adjusted multiple correlation
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Table 1: Graphical Empirical Demand Function (unrestricted)
coefficient ($R^2$), the equations' standard errors ($\hat{\sigma}$), the maximized
values of the log-likelihood function (LLF), the Durbin-Watson
statistic (DW), the Lagrange multiplier statistic for testing against
second order residual autocorrelation ($\chi^2_{SC}(2)$).

The results are in general quite satisfactory: the equations fit
reasonably well, and the value of $R^2$ for the majority of the
industries is well above 0.95. Only in the case of the tobacco
industry does it fall below 0.90. With the exception of the estimates
for industry 31 (Rubber and Plastic Products), the results do not
show significant evidence of residual serial correlation. The
parameter estimates, when statistically significant have signs that are
a priori plausible. The short run elasticities of employment with
respect to real wages and output are generally well determined and
have the correct signs. The (current) real wage variable is
significant at the five percent level in 23 out of the 40 industry
groups, and the (current) output variable is significant in 17 of the
industries. Notice also that the few incorrectly signed estimates
obtained for the real wage and the output variables are not
statistically significant, even at the 10 percent level of significance
using a one-tailed test. Overall the results provide further evidence
in support of the view that both the demand and the product wage
variables are significant determinants of changes in employment,
although, as is already stressed by Peterson (1987), in the case of
most industries changes in demand have been historically more
important than changes in product wages in the explanation of
employment changes.

As far as the time trends are concerned they are significant at
the five percent level only in 12 of the industry estimates, and there are no cases where the coefficient of the time trend is positive and statistically significant. In fact omitting the time trend variable from the analysis in general proved to have only a marginal effect on the coefficient estimates and the significance of the real wage and the output variables. (1)

The results in Table 1 are, however, subject to two important shortcomings: in many cases they seem to be over-parameterized, and the estimates for the industries 16 (Office Machinery etc.), 20 (Ships and other Vessels) and 25 (Tobacco) are unstable. (2) To deal with these shortcomings we estimated a restricted version of the industry employment functions by imposing suitable linear restrictions on the coefficients of (7.1). The coefficient estimates of this 'restricted' specification and their estimated standard errors are summarized in Table 2. The chi-squared statistics for testing the validity of the restrictions together with a number of important diagnostic statistics for tests of misspecification arising from residual serial correlation, functional form, non-normal errors, and heteroscedasticity are given in Table 3. These results are generally more satisfactory than the unrestricted versions. The parameter restrictions cannot be rejected, and only in the case of a very few of the industries do diagnostic statistics indicate that the regression equations are likely to be

---

(1) The effects of omitting the time trend variable on the coefficient estimates were particularly marked only in the case of industries, 2, 3, 5, 10, 24, 32 and 37.

(2) The autoregressive parts of the regressions for these three industries have unstable roots.
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60. Fishing</td>
<td></td>
<td>376,200</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2,515,000</td>
<td>622,000</td>
<td>1,134,600</td>
<td>1,475,200</td>
<td>1,849,000</td>
<td>2,324,200</td>
<td>2,809,400</td>
<td>3,294,600</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 (continued)
<table>
<thead>
<tr>
<th>Industry groups</th>
<th>$\hat{\sigma}^2$</th>
<th>$x^2_T$</th>
<th>$\hat{\sigma}$</th>
<th>$x^2_{SC}(1)$</th>
<th>$x^2_{FF}(1)$</th>
<th>$x^2_{N}(2)$</th>
<th>$x^2_{N}(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Agriculture, forestry and fishing</td>
<td>.9983</td>
<td>0.04(3)</td>
<td>0.0137</td>
<td>0.01</td>
<td>7.25</td>
<td>0.39</td>
<td>2.25</td>
</tr>
<tr>
<td>2. Coal Mining</td>
<td>.9986</td>
<td>3.72(3)</td>
<td>0.0158</td>
<td>0.04</td>
<td>0.91</td>
<td>0.32</td>
<td>0.05</td>
</tr>
<tr>
<td>3. Coke</td>
<td>.9771</td>
<td>5.20(5)</td>
<td>0.0449</td>
<td>0.24</td>
<td>0.67</td>
<td>0.27</td>
<td>1.87</td>
</tr>
<tr>
<td>4. Mineral Oil and Natural Gas</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5. Petroleum Products</td>
<td>.9178</td>
<td>4.89(5)</td>
<td>0.0566</td>
<td>0.48</td>
<td>0.01</td>
<td>1.83</td>
<td>0.85</td>
</tr>
<tr>
<td>6. Electricity Etc.</td>
<td>.9876</td>
<td>2.19(5)</td>
<td>0.0190</td>
<td>0.17</td>
<td>1.26</td>
<td>0.18</td>
<td>0.12</td>
</tr>
<tr>
<td>7. Public Gas Supply</td>
<td>.9719</td>
<td>3.97(4)</td>
<td>0.0322</td>
<td>1.29</td>
<td>0.00</td>
<td>4.86</td>
<td>1.42</td>
</tr>
<tr>
<td>8. Water Supply</td>
<td>.9279</td>
<td>0.73(4)</td>
<td>0.0412</td>
<td>1.67</td>
<td>0.00</td>
<td>0.47</td>
<td>1.05</td>
</tr>
<tr>
<td>9. Minerals and Ores</td>
<td>.9760</td>
<td>2.40(5)</td>
<td>0.0318</td>
<td>1.36</td>
<td>0.16</td>
<td>32.70</td>
<td>0.00</td>
</tr>
<tr>
<td>10. Iron and Steel</td>
<td>.9933</td>
<td>2.49(4)</td>
<td>0.0265</td>
<td>0.08</td>
<td>0.19</td>
<td>1.42</td>
<td>0.43</td>
</tr>
<tr>
<td>11. Non-Ferrous Metals</td>
<td>.9864</td>
<td>2.63(2)</td>
<td>0.0250</td>
<td>0.01</td>
<td>3.47</td>
<td>0.20</td>
<td>1.89</td>
</tr>
<tr>
<td>12. Non-Metallic Mineral Products</td>
<td>.9935</td>
<td>3.44(3)</td>
<td>0.0177</td>
<td>1.11</td>
<td>0.23</td>
<td>0.76</td>
<td>3.15</td>
</tr>
<tr>
<td>13. Chemicals and Manmade Fibres</td>
<td>.9795</td>
<td>6.27(6)</td>
<td>0.0156</td>
<td>3.51</td>
<td>1.80</td>
<td>0.96</td>
<td>1.14</td>
</tr>
<tr>
<td>14. Metal Goods</td>
<td>.9877</td>
<td>2.37(5)</td>
<td>0.0192</td>
<td>0.09</td>
<td>0.27</td>
<td>0.38</td>
<td>1.00</td>
</tr>
<tr>
<td>15. Mechanical Engineering</td>
<td>.9913</td>
<td>3.64(3)</td>
<td>0.0143</td>
<td>0.09</td>
<td>0.10</td>
<td>0.02</td>
<td>0.73</td>
</tr>
<tr>
<td>16. Office Machinery etc.</td>
<td>.8922</td>
<td>10.47(4)</td>
<td>0.0345</td>
<td>0.05</td>
<td>2.68</td>
<td>7.24</td>
<td>5.05</td>
</tr>
<tr>
<td>17. Electrical Engineering</td>
<td>.9698</td>
<td>1.02(2)</td>
<td>0.0173</td>
<td>0.33</td>
<td>0.11</td>
<td>2.19</td>
<td>2.29</td>
</tr>
<tr>
<td>18. Motor Vehicles</td>
<td>.9874</td>
<td>1.29(2)</td>
<td>0.0186</td>
<td>1.55</td>
<td>8.92</td>
<td>3.89</td>
<td>0.01</td>
</tr>
<tr>
<td>19. Aerospace Equipment</td>
<td>.9878</td>
<td>2.21(4)</td>
<td>0.0268</td>
<td>0.90</td>
<td>0.30</td>
<td>1.81</td>
<td>1.30</td>
</tr>
<tr>
<td>20. Ships and other Vessels</td>
<td>.9817</td>
<td>9.70(6)</td>
<td>0.0323</td>
<td>0.45</td>
<td>0.61</td>
<td>0.40</td>
<td>4.46</td>
</tr>
<tr>
<td>21. Other Vehicles</td>
<td>.9973</td>
<td>1.69(4)</td>
<td>0.0241</td>
<td>0.01</td>
<td>0.81</td>
<td>0.17</td>
<td>0.04</td>
</tr>
<tr>
<td>22. Instrument Engineering</td>
<td>.9250</td>
<td>7.92(5)</td>
<td>0.0257</td>
<td>0.47</td>
<td>3.07</td>
<td>0.01</td>
<td>0.84</td>
</tr>
<tr>
<td>23. Manufactured Food</td>
<td>.9837</td>
<td>0.85(3)</td>
<td>0.0164</td>
<td>1.69</td>
<td>2.78</td>
<td>1.33</td>
<td>4.38</td>
</tr>
<tr>
<td>24. Alcoholic drinks etc.</td>
<td>.9232</td>
<td>2.56(4)</td>
<td>0.0269</td>
<td>1.32</td>
<td>0.02</td>
<td>0.94</td>
<td>2.06</td>
</tr>
<tr>
<td>25. Tobacco</td>
<td>.8796</td>
<td>7.09(6)</td>
<td>0.0497</td>
<td>0.25</td>
<td>8.22</td>
<td>0.65</td>
<td>7.62</td>
</tr>
<tr>
<td>26. Textiles</td>
<td>.9981</td>
<td>3.18(5)</td>
<td>0.0175</td>
<td>0.05</td>
<td>4.46</td>
<td>0.74</td>
<td>5.09</td>
</tr>
<tr>
<td>27. Clothing and Footwear</td>
<td>.9994</td>
<td>3.76(6)</td>
<td>0.0110</td>
<td>0.36</td>
<td>1.92</td>
<td>0.62</td>
<td>0.03</td>
</tr>
<tr>
<td>28. Timber and Furniture</td>
<td>.9864</td>
<td>4.24(4)</td>
<td>0.0138</td>
<td>0.00</td>
<td>2.43</td>
<td>1.34</td>
<td>0.30</td>
</tr>
<tr>
<td>29. Paper and Board</td>
<td>.9927</td>
<td>2.86(4)</td>
<td>0.0192</td>
<td>1.09</td>
<td>1.33</td>
<td>1.74</td>
<td>4.41</td>
</tr>
<tr>
<td>30. Books etc.</td>
<td>.9305</td>
<td>4.69(4)</td>
<td>0.0123</td>
<td>1.70</td>
<td>0.01</td>
<td>0.14</td>
<td>0.44</td>
</tr>
<tr>
<td>31. Rubber and Plastic Products</td>
<td>.9813</td>
<td>4.73(4)</td>
<td>0.0173</td>
<td>0.21</td>
<td>1.59</td>
<td>0.96</td>
<td>1.03</td>
</tr>
<tr>
<td>32. Other Manufactures</td>
<td>.9917</td>
<td>7.18(5)</td>
<td>0.0373</td>
<td>0.37</td>
<td>0.21</td>
<td>1.12</td>
<td>0.00</td>
</tr>
<tr>
<td>33. Construction</td>
<td>.9689</td>
<td>5.54(3)</td>
<td>0.0179</td>
<td>5.00</td>
<td>2.34</td>
<td>1.62</td>
<td>1.00</td>
</tr>
<tr>
<td>34. Distribution etc.</td>
<td>.9680</td>
<td>5.44(4)</td>
<td>0.0143</td>
<td>0.49</td>
<td>0.02</td>
<td>0.94</td>
<td>2.06</td>
</tr>
<tr>
<td>35. Hotels and Catering</td>
<td>.9199</td>
<td>3.49(5)</td>
<td>0.0198</td>
<td>0.58</td>
<td>1.88</td>
<td>0.45</td>
<td>0.63</td>
</tr>
<tr>
<td>36. Rail Transport</td>
<td>.9960</td>
<td>2.36(6)</td>
<td>0.0230</td>
<td>0.28</td>
<td>0.00</td>
<td>1.27</td>
<td>1.98</td>
</tr>
<tr>
<td>37. Other Land Transport</td>
<td>.9747</td>
<td>4.04(5)</td>
<td>0.0153</td>
<td>0.02</td>
<td>1.23</td>
<td>0.64</td>
<td>2.71</td>
</tr>
<tr>
<td>38. Sea, Air and Other</td>
<td>.9155</td>
<td>7.87(4)</td>
<td>0.0229</td>
<td>0.27</td>
<td>4.31</td>
<td>0.39</td>
<td>2.75</td>
</tr>
<tr>
<td>39. Communications</td>
<td>.9351</td>
<td>4.42(2)</td>
<td>0.0184</td>
<td>1.56</td>
<td>0.48</td>
<td>0.14</td>
<td>1.81</td>
</tr>
<tr>
<td>40. Business Services</td>
<td>.9940</td>
<td>1.81(5)</td>
<td>0.0128</td>
<td>0.98</td>
<td>2.01</td>
<td>1.98</td>
<td>0.17</td>
</tr>
<tr>
<td>41. Miscellaneous Services</td>
<td>.9812</td>
<td>3.21(6)</td>
<td>0.0222</td>
<td>0.06</td>
<td>0.47</td>
<td>0.39</td>
<td>1.91</td>
</tr>
</tbody>
</table>

Notes:

- $x^2_T$ is the chi-squared statistic for the test of $r$ linear restrictions on the parameters of unrestricted employment equations (see Table 1). The value of $r$ is given in brackets after the statistic.
- $x^2_{SC}(1)$ is the first order LM test of residual serial correlation.
- $x^2_{FF}(1)$ is Ramsey’s RESET test of order 1.
- $x^2_{N}(1)$ is a heteroscedasticity test of order 1.
- $\hat{\sigma}$ is equations’ standard errors.
- $\hat{\sigma}^2$ is the adjusted multiple correlation coefficient.

The underlying regressions and the test statistics reported in this table are computed on Data-FIT package. For details of relevant algorithms and references - see Pesaran and Pesaran (1987).
misspecified. Also note that the restricted estimates for the industries 16, 20 and 25 are no longer unstable, although the equations for the latter two industries are specified in first differences and do not possess long run solutions. The long run elasticities of employment with respect to output and real wages for the 38 industries that do have long run solutions are displayed graphically in figures 1 and 2, respectively.

Although there is still a great deal more room for improving the results by, for example, including 'industry specific' variables in the employment demand functions, we believe that the results obtained so far provide a reasonable basis for the application of the methods developed in this paper to the restricted and unrestricted disaggregate results and those that can be obtained by the direct estimation of the aggregate specification (7.2). For the unrestricted estimate of (7.2) we obtained

\[ (7.3) \quad SLET_t = -136.50 - 0.0217 T_t + 0.5862 \text{SLET}_{t-1} + 0.0819 \text{SLET}_{t-2} \]

\[ + 0.4817 \text{SLYT}_t + 0.0088 \text{SLYT}_{t-1} - 0.3508 \text{SLWT}_t \]

\[ (51.47) (0.0861) (0.2274) (0.0819) \]

\[ - 0.0334 \text{SLWT}_{t-1} + \hat{u}_{tT} \]

\[ (0.0670) (0.1253) \]

\[ LLF = -7.77, \quad R^2 = 0.9958, \quad \hat{\sigma} = 0.3717, \quad DW = 2.06, \quad n = 29, \]

\[ \chi^2_{SC}(1) = 0.88, \quad \chi^2_{FF}(1) = 0.6279, \quad \chi^2_{N}(2) = 4.86, \quad \chi^2_{W}(1) = 1.48. \]

(1) The results in Table 2 are also of some interest in so far as they show evidence of significant aggregate output effects on employment demand at the industry level. See footnote 2 on p. 26.
Figure 1

Histogram of the long run elasticities of employment with respect to output in different industries
Figure 2

Histogram of the long run elasticities of employment with respect to real wages in different industries
The notations are as before, and the test statistics $\chi^2_{SC}$, $\chi^2_{FF}$, $\chi^2_N$, and $\chi^2_H$ are already defined at the foot of table 3. This aggregate specification passes all the tests and has reasonable short run and long run properties. However, it is again over-parameterized. The coefficients of $T_t$, SLET$_{t-1}$, SLYT$_{t-1}$ and SLWT$_{t-1}$ are statistically insignificant whether considered singly or jointly. The chi-squared statistic for the joint test of zero restrictions on the coefficients of these variables was equal to 0.53. So we also estimated the following restricted version of (7.2)

$$\text{SLET}_t = -134.07 + 0.6956 \text{SLET}_{t-1} + 0.4611 \text{SLYT}_t - 0.3718 \text{SLWT}_t + \hat{u}_{tT}$$

with

$$R^2 = 0.9963, \quad \hat{\sigma} = 0.3481, \quad DW = 2.27, \quad n = 29,$$

$$\chi^2_{SC}(1) = 0.65, \quad \chi^2_{FF}(1) = 0.86, \quad \chi^2_N(2) = 5.53, \quad \chi^2_H(1) = 2.20.$$ 

The coefficient estimates are all well determined and imply long run elasticities of aggregate employment with respect to output and real
wages of 1.52 and -1.22, respectively.\(^{(1)}\) The long run real wage elasticity is only marginally different from the value of -0.92 reported recently by Layard and Nickell (1985, p. 177) for the UK. This similarity is especially striking considering the differences that exist between the two analyses as far as the aggregation procedure, the specification of employment function, and the estimation periods are concerned.

We are now in a position to compare the disaggregate and the aggregate results. As far as the in-sample 'predictive' performance of the aggregate and the disaggregate models are concerned, we computed the \(s_{d}^{2}\) criterion [as defined by (4.2)] for the unrestricted and the restricted versions of the disaggregate model. These were 0.1091 and 0.1035 respectively, thus providing evidence of a slightly

\[ SLET_t = -137.01 + 0.6840 SLET_{t-1} + 0.4745 SLYT_t - 0.3830 SLWT_t + \hat{u}_{tT}, \]

\( R^2 = 0.9963, \quad \hat{\sigma} = 0.3487, \quad DW = 2.25, \quad n = 29, \)

\( X_{SC}(1) = 0.56, \quad X_{FF}(1) = 0.07, \quad X_{N}(2) = 4.15, \quad X_{R}(1) = 2.18, \)

which differ only marginally from the OLS results. In fact the Wu-Hausman statistic (\( T_2 \) statistic in Wu (1973)), for the test of the 'exogeneity' of SLYT\(_t\) and SLWT\(_t\) in (7.4), using \( \xi_t \) as the instruments, was equal to 0.112, which is well below the 5 percent critical value of the \( F \) distribution with 2 and 23 degrees of freedom.
better fit for the restricted version of the disaggregate model.\(^{(1)}\)

The value of the goodness-of-fit criterion for the aggregate equations (7.3) and (7.4) were equal to 0.1359 and 0.1153, respectively. These results are summarized in Table 4, where the uncorrected GG criterion (the first term on the right hand side of (4.8)) is also reported in brackets. On the basis of the proposed choice criterion the restricted as well as the unrestricted versions of the disaggregate model are preferable to the aggregate equation. The computation of the statistic for the test of perfect aggregation defined by (5.8) also provided additional support in favour of the disaggregate model. In the case of the unrestricted version the value of this test statistic was equal to 81.66, which is approximately distributed as a \(X^2_{29}\), thus firmly rejecting the hypothesis of perfect aggregation. This is also clearly reflected in the estimates of the long run elasticities obtained from the disaggregate and the aggregate results. For example, concentrating on the restricted versions of the employment functions, the long run elasticity of aggregate employment with respect to output based on the disaggregate results (Table 2), turned out to be equal to 0.724 as compared with the figure of 1.52 obtained using the aggregate

\[(1)\text{ Notice that in general there is no reason to believe that the restricted model should perform better than the unrestricted model as far as the } s_d^2 \text{ criterion is concerned. Although it is true that the imposition of statistically 'acceptable' linear restrictions on the parameters of the micro equations, such as omitting one or more variables from the micro equations whose } t-\text{ or } F- \text{ values are less than unity, lowers the estimates of } \sigma_{ij}, \text{ the same is not true of the estimates of the contemporaneous covariances, } \sigma_{ij}, i \neq j. \text{ As a result the effect of restrictions on }\]

\[s_d^2 = \sum_{i=1}^{m} \hat{\sigma}_{ii} + 2 \sum_{i>j} \hat{\sigma}_{ij}\]

will, in general, be ambiguous.
Table 4

Relative predictive performance of the aggregate and the disaggregate employment functions (1956-1984)

<table>
<thead>
<tr>
<th></th>
<th>Aggregate equations</th>
<th>Disaggregate equations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unrestricted</td>
<td>Restricted</td>
</tr>
<tr>
<td>All industries</td>
<td>0.1382</td>
<td>0.1201</td>
</tr>
<tr>
<td></td>
<td>(0.0846)</td>
<td>(0.0859)</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.0492</td>
<td>0.0433</td>
</tr>
<tr>
<td></td>
<td>(0.0439)</td>
<td>(0.0373)</td>
</tr>
</tbody>
</table>

1. See equations (7.3) and (7.5).
2. See equations (7.4) and (7.6).
3. See the results in tables 1 and 2.
4. Excluding Industry 4, Mineral Oil and Natural Gas.
5. Industries 10 to 32 inclusive.

Bracketed figures refer to the degrees-of-freedom uncorrected measure of the choice criterion, given by the first term in the expression for $s_d^2$ defined in (4.6).
specification (7.4). Similarly the long run elasticity of aggregate employment with respect to real wages based on the disaggregate specification was equal to -0.4551 as compared with the estimate of -1.22 based on the aggregate specification (7.4). These results clearly suggest the existence of important upward bias in the estimates of output and real wage elasticities of employment demand obtained in the literature using an economy wide aggregate specification.

7.2 Results for the manufacturing industries

Having rejected the aggregate employment function in favour of the disaggregate model, the question of what the appropriate level of disaggregation should be naturally arises. One possibility would be to repeat the above analysis for all possible levels of disaggregation. Here in the way of illustration we only consider the problem in the case of the manufacturing industries. The disaggregate results for this industry grouping are given by the industries labelled 10 to 32 inclusive in Tables 1 and 2. We also obtained the following estimates of the unrestricted and the restricted employment demand functions for the manufacturing sector as a whole:

$$SLEM_t = -65.58 - 0.0039 T_t + 0.7491 SLEM_{t-1} - 0.0162 SLEM_{t-1}$$

$$+ 0.4933 SLYM_t - 0.0897 SLYM_{t-1} - 0.2979 SLWM_t$$

$$- 0.0148 SLWM_{t-1} + \hat{u}_t$$

(7.5)

(1) The estimates of the long run elasticities for the disaggregate model were computed from the simple averages of the micro coefficients.
\[ \text{LLF} = 7.20, \quad R^2 = 0.9968, \quad \hat{\sigma} = 0.2218, \quad DW = 2.21, \quad n = 29, \]
\[ x_{SC}^2(1) = 4.56, \quad x_{FF}^2(1) = 0.01, \quad x_N^2(2) = 0.24, \quad x_{H}^2(1) = 1.70, \]

and

\[ \text{LLF} = 7.13, \quad R^2 = 0.9972, \quad \hat{\sigma} = 0.2080, \quad DW = 2.19, \quad n = 29, \]
\[ x_{SC}^2(1) = 0.34, \quad x_{FF}^2(1) = 0.00, \quad x_N^2(2) = 0.21, \quad x_{H}^2(1) = 1.47, \]

where
\[ SLEM_t = \sum_{i=10}^{32} LEH_{it}, \quad SLYM_t = \sum_{i=10}^{32} LY_{it}, \quad SLWM_t = \sum_{i=10}^{32} LW_{it}. \]

The restricted version (7.6) clearly cannot be rejected against the unrestricted version (7.5). In this application the values of the goodness-of-fit criterion \( s^2_d \) for the unrestricted and the restricted models were 0.0506 and 0.0455 respectively, indicating that the restricted version of the disaggregate model has a better in-sample performance in so far as predicting the aggregate employment variable \( SLEM_t \) is concerned. The goodness-of-fit criterion for the aggregate specifications (7.5) and (7.6) are given by 0.0492 and 0.0433 respectively. (See also Table 4). Hence on the basis of the choice criterion, for the manufacturing industries the

---

(1) We also estimated the restricted version (7.6) by the IV method using \( z_t = (1, SLEM_{t-1}, SLEM_{t-2}, SLYM_{t-1}, SLYM_{t-2}, SLWM_{t-1}, SLWM_{t-2}) \) as instruments and obtained very similar results.
aggregate models give marginally a better fit than either of the
disaggregate models. This, of course, does not mean that the
aggregate model is not subject to the aggregation error problem. In
fact the application of the test of perfect aggregation to this example
resulted in the value of 69.92 for the $\bar{a}_m$ statistic which is well in
excess of the 5% critical value of the $\chi^2$ distribution with 29 ($= n$)
degrees of freedom.

The rejection of the perfect aggregation hypothesis is also
reflected in the large differences that exist between the estimates of
the long run real wage and output elasticities of the manufacturing
employment based on the disaggregate and the aggregate results. In
the case of the restricted models, the estimates of the long run real
wage elasticity based on the aggregate and the disaggregate models
were $-1.21$ and $-0.509$, respectively. The corresponding figures for
the long run real output elasticities were 1.54 and 0.763, respectively.
The better performance of the aggregate model should be interpreted
as an important indication that the disaggregate employment functions
are misspecified. This suggests the need for a much more detailed
analysis of employment demand at the industry level, which may
involve including 'industry specific' variables in employment
equations, experimenting with a different choice of functional forms
across industries, or searching for new industry-specific explanatory
variables, or even compiling a more reliable set of micro-data.

8. Concluding remarks

In this paper our primary concern has been with the problem of
choice between macro and micro regression equations for the purpose
of predicting macro variables. The test of perfect aggregation developed in the paper also addresses the macro prediction problem; although as our application to the UK employment demand functions shows, it has some bearing on the problem of aggregation bias as well. In using the goodness-of-fit criterion and the test of perfect aggregation it is, however, important to note that these methods, like most other methods of inference in econometrics, suffer from the fact that they may have little to say on the validity of the aggregation conditions outside the estimation period. In the case of aggregation across micro units this problem is especially serious as the extension of the results of aggregation tests to the post estimation period requires stability of the micro-coefficients as well as the stability of the industrial composition of the economy.
Appendix A

A proof of the asymptotic validity of the proposed test of perfect aggregation

In this appendix we provide a proof of the theorem stated in the paper. Let

$$(A1) \quad g_m = \left( \sum_{i=1}^{m} \hat{\sigma}_{ii} H_i \right)^{-1} \left( e_a - e_d \right),$$

where $e_a$, $e_d$, $H_i$ and $\hat{\sigma}_{ii}$ are already defined in the text. For convenience we reproduce them here

$$e_a = [I_n - X_a (X_a' X_a)^{-1} X_a'] y_a = (I_n - A_a) y_a,$$

$$e_d = \sum_{i=1}^{m} M_i y_i; \quad M_i = I_n - X_i (X_i' X_i)^{-1} X_i' = I_n - A_i,$$

$$\hat{\sigma}_{ii} = y_i' M_i y_i / (n - k),$$

$$H_i = A_i - A_a.$$

Then the test statistic in the theorem can be written as

$$(A2) \quad a_m = g_m' g_m.$$

Consider now the probability limit of $\tilde{\psi}_m = m^{-1} \sum_{i=1}^{m} \hat{\sigma}_{ii} H_i$ as $m \to \infty$. Under (2.1) we obtain
(A3) \[ \hat{\psi}_m = [m(n - k)]^{-1} \sum_{i=1}^{m} (u_i'M_iu_i) H_i^2. \]

But since \( M_i \) is an idempotent matrix of rank \( n - k \), we can also write

(A4) \[ \sigma_{ii}^{-1} u_i'M_iu_i = \sum_{t=1}^{n-k} \epsilon_{it}^2, \quad i = 1, 2, \ldots, m \]

where \( \epsilon_{it} \) represent scalar random variables distributed independently across \( i \) and \( t \) with zero means and unit variances. Substituting (A4) in (A3) yields

(A5) \[ \hat{\psi}_m = (n - k)^{-1} \sum_{t=1}^{n-k} \left( \sum_{i=1}^{m} \sigma_{ii} \epsilon_{it}^2 H_i^2/m \right). \]

But, noting that \( H_i = A_i - A_a \), we have

(A6) \[ m^{-1} \sum_{i=1}^{m} \sigma_{ii} \epsilon_{it}^2 H_i^2 = f_m A + F_m - F_m A - A F_m, \]

where

\[ f_m = m^{-1} \sum_{i=1}^{m} \sigma_{ii} \epsilon_{it}^2, \]

\[ F_m = m^{-1} \sum_{i=1}^{m} \sigma_{ii} \epsilon_{it} A_i. \]

Now under assumption 4 it readily follows that

\[ \liminf_{m \to \infty} (f_m - \tau^2) \lim_{m \to \infty} \left( \sum_{i=1}^{m} \epsilon_{it}^2 \right). \]
and since $\varepsilon_{it}$ are identically and independently distributed random variables, then by the law of large numbers $m^{-1} \sum_{i=1}^{m} \varepsilon_{it}^2 \to 1$, and

\[ \lim_{m \to \infty} \frac{\sum_{i=1}^{m} \varepsilon_{it}^2}{m} = 1. \]

Similarly, under assumptions 3 and 4 we have

\[ \lim_{m \to \infty} \left( \sum_{i=1}^{m} \sigma_{ii} \varepsilon_{it}^2 \right) = r^2P < \infty, \]

where $P$ is already defined by assumption 3. The results (A7) and (A8) establish the existence of the probability limits of $f_m$ and $F_m$, as $m \to \infty$, and this in turn establishes (using (A6) and noting that by assumption 2 matrix $A_a$ has a finite limit as $m \to \infty$) that

\[ \lim_{m \to \infty} \sum_{i=1}^{m} \sigma_{ii} \varepsilon_{it}^2 H_i^2 = \lim_{m \to \infty} \sum_{i=1}^{m} \sigma_{ii} H_i^2. \]

Using this result in (A5) we finally obtain

\[ \hat{\psi}_m = m^{-1} \sum_{i=1}^{m} \hat{\sigma}_{ii} H_i^2 \quad \text{and} \quad \lim_{m \to \infty} \sum_{i=1}^{m} \sigma_{ii} H_i^2 = \psi. \]

Therefore, asymptotically we have\(^{(1)}\)

\[ g_m \prec \psi - \frac{1}{m} \left( e_a - e_d \right). \]

But under (2.1) on the assumption that $H_\phi, \sum_{i=1}^{m} X_i \beta_i = X_a \beta$ holds

---

\(^{(1)}\) Note that by assumption 5, matrix $\psi$ is non-singular.
\[ m^{-H} (e_a - e_d) = m^{-H} \sum_{i=1}^{m} H_i u_i. \]

Hence

(A10) \[ g_m \overset{d}{=} m^{-H} \sum_{i=1}^{m} z_i, \]

in which

\[ z_i = \sigma_{ii}^{-H} \psi^{-H} H_i z_i, \]

and \( \psi_i = u_i / \sqrt{\sigma_{ii}} \). We now show that under the assumptions of the theorem, as \( m \to \infty \), the sum \( s_m = m^{-H} \sum_{i=1}^{m} z_i \) tends to a multivariate normal distribution with mean zero and the covariance matrix \( I_n \); an identity matrix of order \( n \). For this purpose it is sufficient to demonstrate that for any fixed vector \( \lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)' \), the limiting distribution of \( \lambda' s_m \) is \( N(0, \lambda' \lambda) \).

Let

(A11) \[ d_m = \lambda' s_m = m^{-H} \sum_{i=1}^{m} w_i, \]

in which

(A12) \[ w_i = \sigma_{ii}^{-H} \psi^{-H} H_i v_i, \quad i = 1, 2, \ldots, m \]

is now a scalar random-variable. We have, for all \( i \),

\[ E(w_i) = 0, \]
\[ V(\omega_i) = \sigma_{ii} \lambda^{\psi^{-1} H_1^2 \psi^{-1} \lambda} . \]

Setting \( \psi = \psi^{-1} H_2 \), then

\[ c_i \mathcal{X}_m = \sum_{i=1}^{m} V(\omega_i) = \mu'(\sum_{i=1}^{m} \sigma_{ii} H_1^2) \mu . \]

Denoting the \( (t, t') \) element of matrix \( H_1 \) by \( h_{it,tt'} \), we also have [using (A12)]

\[ w_i = \sum_{t=1}^{n} \left( \sum_{t=1}^{n} \mu_t h_{it,tt'} \right) v_{it'} . \]

Therefore, since by assumption \( \psi \) is non-singular and \( h_{it,tt'} \) are bounded in absolute value for all \( i \), then

\[ |w_i| \leq n \kappa \sigma_{ii}^\psi \sum_{t=1}^{n} |v_{it'}| , \]

where \( |\mu_t h_{it,tt'}| < \kappa < \infty \). Consequently

\[ E|w_i|^3 \leq n^3 \kappa^3 \sigma_{ii}^\psi^2 \sum_{t=1}^{n} |v_{it}^3| . \]

However, since the random variables \( v_{it} \) are i.i.d. with finite third order moments, then \( E|\sum_{t=1}^{n} v_{it}|^3 \leq n \sigma^3 \), where \( \sigma^3 = E|v_{it}|^3 \), and

\[ E|w_i|^3 \leq n^4 \kappa^2 \sigma_{ii}^\psi^2 \sum_{t=1}^{n} |v_{it}|^3 . \]

We are now in a position to apply the Lyapunov Central Limit Theorem
to the sum $d_m$ defined by (A11). Setting

$$B_m^3 = \sum_{i=1}^{m} \mid E \mid w_i \mid^3,$$

then using (A14) it follows that

$$B_m^3 \leq \left( \sum_{i=1}^{m} \sigma^{-3/2}_{ii} \right)^{3/2},$$

which together with (A13) yields (2)

$$\lim_{m \to \infty} \left[ \frac{B_m}{C_m} \right] \leq \left\{ \frac{n^{4/3}}{\tau} \right\} \lim_{m \to \infty} \left[ \sum_{i=1}^{m} \sigma_{ii}^{3/2} \right]^{1/3}.$$ 

But under assumption 4

$$\lim_{m \to \infty} \left[ \sum_{i=1}^{m} \sigma_{ii}^{3/2} \right]^{1/3} \leq \lim_{m \to \infty} (m^{-1/6} \tau) = 0,$$

and for a fixed $n$, we have $\lim(B_m/C_m) = 0$, as $m \to \infty$; and the condition of the Liapunov theorem will be met. Hence

$$a_m a_m = \tilde{N}(0, I_n).$$

Now using (A2) we have

$$a_m = \frac{s_m}{a_m} \chi_n^2.$$ 

Q.E.D.

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(1) See, for example, Rao (1973, p. 127)

(2) Notice that $\lim (m^{-1} \sum_{i=1}^{m} \sigma_{ii} H_i^2 y_i) = \mu' \Sigma y = \lambda \tilde{\lambda}$. 
Data Appendix: Data Sources and Definitions

The data used in the empirical analysis in section 7 are annual observations on 41 industry groups for the UK obtained from the Cambridge Growth Project Databank. The data on industry man-hours, employment, wages and salaries and employers' contributions were originally provided by the Institute for Employment Research at the University of Warwick. The data on industry output were obtained from the Central Statistical Office's commodity flow accounts adjusted for our industrial classification. The data on producer price indices of industry output were obtained from a number of published sources including the Department of Trade and Industry's publication 'British Business', the CSO's publications, the 'Annual Abstract of Statistics' and the 'Monthly Digest of Statistics', and the Department of Energy's 'Energy Trends'.

Some of the 41 industry groups are identical to the 'groups' distinguished in the 1980 Standard Industrial Classification. However in view of the significant differences between them in a large number of cases, the groups are listed in Table B1, using as a reference the Division, Class or group of the 1980 Standard Industrial classification. In the analysis of the manufacturing sector groups 10 to 32 inclusive are included.

For empirical estimation, the man-hours employed (EH_t) are defined as a product of the actual hours worked per week and the numbers employed in each of 41 industries, including self employed ('000s) in these industries. Industry output (Y_t) is gross value added by industry in 1980 prices (£m). Average real wage rate (W_t) is a measure of the real product wage by industry. It is obtained
by first deflating an industry’s total labour costs including both employees’ wages and salaries and employers’ national insurance contributions (£m) by the price index of industry output (1980 = 1.00). This is then divided by the man-hours employed in that industry to obtain the average real wage rate.

All the data are annual covering the period 1954-1984 with both the aggregate and disaggregate equations estimated over the period 1956-1984. These data, and the computer programmes used both in estimation and in the computation of the choice criterion and the statistics for the test of perfect aggregation, are available on request from the authors.
<table>
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<tr>
<th>Industry</th>
<th>Division, class or group</th>
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