FEDERAL LENDING AND THE MARKET FOR CREDIT

by

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UCLA Department of Economics
Working Paper #504
Revised November 1988
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ABSTRACT

Federal lending is an important, but unrecognized feature of modern credit markets. This paper analyzes the effects of credit interventions on credit allocation and economic efficiency. The underlying model posits asymmetric information between borrowers and lenders, and allows for market clearing, rationed, or redlined equilibria. Principal results include: unsubsidized credit interventions are neutral; the effectiveness of alternative lending instruments depends on whether rationing occurs; interactions among programs can have perverse, unintended effects; and, even without an informational advantage, the government may enact welfare-improving credit policies under certain conditions.

*This paper is based on Chapter 3 of my Ph.D. dissertation. An earlier version circulated under the title, "Federal Lending Policies and the Market For Credit" (November 1986). I would like to thank my dissertation committee, Michael Boskin, Doug Bernheim, and John Shoven for many helpful discussions. I have also benefitted from conversations with Brad Barham, Al Harberger, Seonghwan Oh, John Riley, Russell Roberts, John Karl Scholz, and especially David Butz and Michael Waldman. I thank the John M. Olin Foundation for financial support, and Lorraine Grams and Kathy Martija for word processing assistance.
I. Introduction

The federal government is a major participant on both sides of the capital market. Although federal borrowing has been the subject of heated debate in recent years, relatively little attention has been given to the government's longstanding and sizable presence as a lender. The government runs dozens of direct loan and loan guarantee programs. Government-Sponsored Enterprises and tax-exempt status assist many additional borrowers. Taken together, federal programs have directly subsidized, guaranteed, or extended approximately one-third of all net credit issued to non-federal sectors since 1980.

This paper analyzes the effects of such policies on credit allocation and economic efficiency in a model where borrowers retain private information concerning their ability to repay loans. Although models of asymmetric information in credit markets have proliferated in recent years, only Mankiw [1986] and Smith and Stutzer [1988] have studied credit subsidies in this framework.¹

In Mankiw [1986], lenders can supply funds to a risk-free investment, or to a continuum of risky investors. Mankiw shows that small changes in the risk-free rate can generate large shifts in credit allocation. In addition, credit subsidies can, under certain conditions, raise welfare, because the ordering of projects by social return and by bank return varies. Smith and Stutzer develop a credit market model based on Rothschild and Stiglitz [1976]. They show that the effects of alternative interventions depend on how the policy affects the self-selection constraint. In their model, loan guarantees are Pareto-improving because they reduce the severity of the self-selection constraint. Other policies face a tradeoff between raising overall welfare or raising welfare of the rationed borrowers.
These papers show that in the presence of asymmetric information, credit policy can generate important, nonstandard effects on allocation and efficiency. This paper examines similar issues, but differs from the papers above in several ways. First, unlike Mankiw, the paper explicitly models the effects of alternative lending instruments. Since the government can choose to intervene through any of a variety of policies, the differing effects of each instrument are important. Second, unlike Smith and Stutzer, the model examines regimes characterized by market clearing, rationing, or redlining of the target group. Since many recipients of credit assistance are thought to be rationed or redlined without credit subsidies, it is important to examine credit policy in different states of the market. Third, unlike either paper, interactions among programs that target different groups are examined. Given that the government simultaneously assists many groups of borrowers, these interactions are an important policy consideration. Fourth, the resource costs of federal credit programs are explicitly included in the welfare calculations. By modelling these features of federal credit policy, the paper generates several new results, as described below.

The next section provides an overview of federal lending activity. In Section III, I develop a model of the credit market. The model allows for many identifiable borrower groups. Group identity is a (noisy) public signal of borrower riskiness. Each borrower retains private information concerning the riskiness of his projects. Equilibrium may be characterized by market clearing, rationing, or redlining of target groups. The key feature of the equilibrium is that banks order observable borrower groups by the maximum bank return to lending to each group, and serve the groups sequentially. The comparative statics of market clearing and rationed equilibria are
contrasted. Section IV incorporates credit subsidies and their funding requirements into the model. Three types of subsidies are modeled: direct loans, loan guarantees, and pure interest subsidies.

Section V examines the allocational effects of credit subsidies. Unsubsidized credit policies are shown to be neutral. In the market clearing equilibrium, credit subsidies operate in standard ways. However, in the rationed equilibria, the effectiveness of interest subsidies (as opposed to loan guarantees) is severely curtailed. This occurs because rationing is effectively an unwillingness of banks to lend, rather than an unwillingness to pay on the investors' part. Thus, policies that serve only to reduce borrower payments do not attack the source of the rationing constraint.

Interactions among programs are shown to be potentially very important. Specifically, by subsidizing particular groups, the government can change the banks' ordering of borrower groups by maximum return. If the borrowers eliminated from the credit market by such reorderings are also target groups, then the government is essentially competing against itself in attempting to fund many groups. That is, credit programs may serve only to rearrange funds among target groups, rather than increase total targeted lending. More generally, as subsidies to any one group increase, the government must raise its subsidies to other groups to maintain the latter's initial credit allocation.

Welfare results are developed in Section VI. In the absence of asymmetric information, the market funds all socially efficient projects and only those projects. When private information is introduced, however, some projects with high expected total return will not be funded. This occurs because, as in Mankiw [1986], the ordering of projects by expected bank return and expected social return differs, in general. As a result, small credit subsidies can improve on the market outcome under certain conditions.
Section VII shows that the basic framework can accommodate collateral requirements. The last section discusses some caveats and extensions of the results.

II. Federal Lending Activity

The federal government has maintained a long-standing role as a supplier of credit. Saulnier, et al. (1958) present a thorough description of credit programs prior to 1950. These programs focused almost exclusively on housing, agriculture, and business. As early as 1952, federal credit accounted for 15% of total credit market debt. This figure rose gradually to 31% in 1982 and has since fallen to 29% in 1986. This secular rise in credit volume has been matched by similar increases in effective subsidy rates and applications of federal credit.

Hardin and Denzau (1981) report the existence of more than 350 direct and guaranteed lending programs. In 1987, the government issued $30 billion worth of new direct loans. These loans are concentrated in the agricultural and rural sectors and typically offer large subsidies. Smaller direct lending programs support defense, commerce, energy, housing, and transportation.

Primary and secondary loan guarantees accompanied $142 billion and $140 billion, respectively, of new credit in 1987. Most loan guarantees target the mortgage market, but guarantees also play important roles for small businesses, students, and exporters. In addition, the government has intervened on several occasions to guarantee the debt of large concerns after they experienced financial crises. In recent years, these emergency guarantees have assisted Chrysler, New York City, and Lockheed. Finally, loan guarantees have supported a variety of special projects, such as energy development. Thus, loan guarantees have become a popular instrument for a
variety of policy purposes.\textsuperscript{3}

Government-Sponsored Enterprises assist borrowers in housing, agricultural, and student loan markets, primarily by providing secondary markets.\textsuperscript{4} In 1987, new obligations of these Enterprises equalled $427 billion. Tax-exempt status allows state and local governments to reduce the cost of their own projects and to run their own lending programs by passing on the interest savings to preferred borrowers. In 1987, $100 billion worth of new tax-exempt debt was issued.

The overall magnitude of recent federal and federally assisted lending is shown in Table 1. Since 1980, the net credit advanced through federal programs is approximately the same magnitude as federal borrowing. Notably, this figure represents approximately one-third of all borrowing by non-federal sectors.

However, as shown below, credit volume is a poor indicator of a program's effects, which depend instead on effective subsidy rates and other factors. Subsidies vary considerably across programs. For groups such as FHA mortgage borrowers, the provision of credit is on nearly market terms. For others, including students, farmers, and small businesses, government credit provides effective price reductions of 20\% to 40\%.

Finally, it should be noted that credit interventions are not unique to the United States. Rosen [1981] and Woodhall [1978] document the credit assistance given to homeowners and students, respectively, in selected European countries.
<table>
<thead>
<tr>
<th>Description</th>
<th>$ (Billion)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Credit Advanced in Nonfinancial Credit Markets(^a)</td>
<td>4,782</td>
</tr>
<tr>
<td>Net Federal and Federally Assisted Lending</td>
<td>1,208</td>
</tr>
<tr>
<td>Direct Loans</td>
<td>116</td>
</tr>
<tr>
<td>Loan Guarantees</td>
<td>251</td>
</tr>
<tr>
<td>Government-Sponsored Enterprises</td>
<td>442</td>
</tr>
<tr>
<td>Tax-Exempt Borrowing</td>
<td>399</td>
</tr>
<tr>
<td>Federal Borrowing From the Public</td>
<td>1,253</td>
</tr>
<tr>
<td>Federal Lending as a percentage of</td>
<td></td>
</tr>
<tr>
<td>Net Credit</td>
<td>25</td>
</tr>
<tr>
<td>Federal Borrowing</td>
<td>96</td>
</tr>
<tr>
<td>Nonfederal Borrowing</td>
<td>34</td>
</tr>
</tbody>
</table>

\(^a\)Net lending is new lending minus repayments.

III. A Model of the Credit Market

A. Description

I assume the existence of a credit market with many borrowers, depositors, and financial intermediaries. The model is related to Stiglitz and Weiss [1981] in structure, and examines a competitive loan market characterized by asymmetric information between borrowers and lenders. All agents are risk-neutral, and there is assumed to be no aggregate risk. Depositors each own a unit of wealth, but have no investment projects. They supply funds to the banking system according to a function $S(\rho)$, where $\rho$ is the certain rate of return on bank deposits.

Borrowers are divided into 2 groups: the target group (T) for federal lending and non-targeted, general (G) borrowers. These groups are observationally distinguishable, thus allowing banks and the government to identify eligible borrowers for credit subsidies, and providing a (noisy) signal of borrower riskiness, as described below.

Within each group $I$ (I=G,T), borrowers are indexed by $j$ and distributed with density $f_I(j)$ along the interval $[0,1]$. A borrower's location $j$ within the group is known only to that borrower, and will provide the remaining information concerning the riskiness of his investment.

These assumptions are meant to provide a stylized way of partitioning information sets into public and private components. Banks operate in two distinct, but related loan markets with unobservable borrower diversity in each. Thus, banks set different loan contracts for each group. Loan contracts will consist of only an interest rate; adding a collateral requirement is discussed in Section VII.

Each borrower has one available project of unit size but must borrow in order to undertake the investment. The project undertaken by borrower
(I,j) succeeds with probability \( p_I(j) \), in which case the gross return is \( R_I(j) \). Otherwise, the gross return is zero. If the project succeeds, the loan is repaid.

Within each group, borrowers are ordered such that as \( j \) rises, \( p_I(j) \) rises and \( R_I(j) \) falls. Thus, within each group, higher \( j \) values represent relatively safer projects which, if successful, have relatively lower returns. These relations are depicted in Figure 1. The total expected return, \( \rho_I \), can vary with \( j \).

Borrowers maximize expected profits, given by

\[
(1) \quad E\Pi_I(j) = p_I(j)(R_I(j) - r_i)
\]

where \( r_i \) is one plus the interest rate on loans to group \( I \). Borrower \((I,j)\) accepts a loan only if \( R_I(j) \geq r_i \). Define \( j_I^* \) as the marginal borrower in group \( I \). Then:

\[
(2) \quad R_I(j_I^*) = r_i
\]

and only borrowers riskier than \( j_I^* \) (those with \( j < j_I^* \)) choose to borrow. Loan demand for group \( I \) is given by

\[
(3) \quad L^D_I(r_i) = \int_0^{j_I^*} f_I(j) dj = F(j_I^*)
\]

and is downward sloping because \( \frac{\partial j_I^*}{\partial r_i} < 0 \).

The expected gross return to the bank on loans to group \( I \) is given by:

\[
(4) \quad \rho_I = \phi_I(r_i) \cdot r_i = \phi_I(r_i),
\]

where \( \phi_I \) is the overall probability of repayment and is given by
Figure 1

Description of Borrowers and Projects

Figure 2

Banks' Expected Return to Lending
\[
\phi_I(r_I) = \frac{\int_0^{j^*(r_I)} p_I(j) f_I(j) dj}{F_I(j^*(r_I))}.
\]

Differentiation of (5) yields \( \frac{\partial \phi_I}{\partial r_I} < 0 \): an increase in interest rates reduces the overall probability of repayment.\(^8\) This occurs because increases in interest rates crowd out those with the lowest \( R_I(j) \); that is, they eliminate the safest borrowers in the market. Thus, each sectoral market is characterized by adverse selection among loan applicants. Therefore, the expected bank return may rise or fall with increased interest rates.

Figure 2 shows three features of the \( \rho_I \) curves used in the analysis below. First, for each group, the expected bank return is assumed to rise to an interior maximum and then fall in response to further increases in interest rates. This feature will allow for the possibility of, but does not impose, rationing. Second, the maximum bank return on general loans \( \rho_G(\bar{r}_G) \) is assumed to be greater than that on target loans \( \rho_T(\bar{r}_T) \). As a consequence, in some situations, banks will not lend any funds to the target group, or will ration the target group, while the market for general loans clears. Third, \( \rho_G(r) > \rho_T(r) \) for all \( r \).\(^9\) As a consequence, in equilibrium, general borrowers face lower interest rates and default less frequently than target borrowers. Although it may be defended on grounds of realism, this assumption is made here for convenience only; changing the assumption would have no important effect on the main results.

B. Equilibrium

The model is competitive in that there are many agents of each type and entry is free. There is, however, an important departure from the standard competitive framework: in addition to caring about total demand, banks also
care about the identity of the borrower.

Let $S_I$ be the aggregate supply of loans from banks to group $I$. Equilibrium is defined as a set of interest rates and credit allocations such that all agents optimize given the choices of others. Formally, equilibrium is given by $(\rho^*, r_I^*, S_I^*)$ for $I = G, T$, such that:

(6) if $S_I^* > 0$ then $\rho^* = \rho_I^*(r_I^*)$

(7) $S_I^* \leq L_I^D(r_I^*)$

(8) if $0 < S_I^* < L_I^D(r_I^*)$ then $r_I^* = \bar{r}_I$, and

(9) $S(\rho^*) = S_G^* + S_T^*$.

Equation (6) requires that banks earn an expected profit of zero on all loans that they issue. This result follows from constant returns to scale and free entry in banking. Equation (7) is a feasibility condition: the actual volume of credit accepted by each group can not be larger than the quantity demanded by that group.

Equation (8) states that rationing can occur only at the interest rate that maximizes the banks' rate of return on loans to a group ($\bar{r}_I$ in Figure 2). If a group were rationed at some lower interest rate, banks could raise their expected rate of return by raising interest rates, and previously rationed borrowers would accept those higher rates. Thus, rationing can occur in equilibrium only if $r_I^* = \bar{r}_I$.10 Equation (9) stipulates that no idle funds exist. Equations (6)-(9) are necessary and sufficient conditions for equilibrium.

Depending on the relative magnitudes of aggregate credit supply and demand, equilibrium can entail redlining, rationing, or market clearing status for the target group. To show this, it is useful to construct an effective demand curve, as shown in Figure 3. In Figure 3, loan demand is
**Figure 3**

*Effective Demand*

**Figure 4**

*Types of Market Equilibrium*
plotted in the lower right quadrant and banks' expected return in the upper left. Effective demand, shown in the upper right quadrant, refers to demand by a borrower group that banks are willing to supply, given the cost of funds. Thus, the effective demand for loans by group \( I \) is

\[
L^I = \begin{cases} 
0 & \text{if } \rho^* > \hat{\rho}^I \\
L^D_I & \text{if } \rho^* < \hat{\rho}^I.
\end{cases}
\]

For example, in Figure 2, if \( \rho^* > \hat{\rho}^T \), banks will not consider making loans to the target group, since such loans would generate negative expected profits. If \( \rho^* = \hat{\rho}^I \), effective demand for group \( I \) is determined as a residual, after other credit demands have been met, such that \( 0 \leq L^I \leq L^D_I(\check{r}^I) \).

Summing over borrower groups yields an aggregate effective demand curve, as shown in Figure 4.\(^{11}\) The nature of equilibrium then depends on the placement of the aggregate supply of funds: \( S_1, S_2 \) and \( S_3 \) represent market-clearing, rationing, and redlining outcomes for the target group. The market for general loans clears in all three cases.\(^{12}\) If the supply of funds were infinitely elastic, no group would be rationed, but some could still be redlined, depending on the level of \( \rho^* \).

C. Properties

Three properties of the model deserve comment at this point. First, in all equilibria, banks satisfy general demand first. When the target group is redlined, this proposition is obvious, since only general borrowers receive funds. In market clearing equilibria, banks meet all demand, so the order in which they do so does not matter. However, in the rationing equilibrium, the "general borrowers first" result dictates that target borrowing is determined as a residual.
The intuition for this result warrants emphasis. If targeted borrowers are rationed, and banks did not satisfy all general demand first, there would be unmet general demand at some $\hat{r}_G < \hat{r}_G$ in Figure 2. Some of the rejected general applicants would be willing to borrow at a higher interest rate, which would give the bank a higher expected return than that available on target group loans. Therefore, banks that do not lend to general borrowers first are passing up profitable opportunities. More generally, with many borrower groups, banks order the groups by maximum rate of return available ($\hat{\rho}_I$) and serve the groups sequentially. All groups with $\hat{\rho}_I > \rho^*$ have clearing credit markets, those with $\hat{\rho}_I < \rho^*$ are redlined, and those with $\hat{\rho}_I = \rho^*$ are rationed. This result will have important implications for federal credit interventions.

Second, comparative static properties differ significantly across the market clearing and rationing regimes. For example, consider a small leftward shift in the supply schedule. In the market clearing case, both loan rates rise and both $S_I$ fall. In contrast, in the presence of rationing, all interest rates and general demand are unaffected. The entire effect is reflected in a fall in targeted lending. Thus, shifts in aggregate level of credit can have important effects on the allocation of credit. As a second example, an outward shift in target loan demand (shown by the dotted line in Figure 4) will have no effect on a rationed equilibrium, but will raise all rates and shift the allocation and level of credit in the market clearing equilibrium. Credit subsidies thus may be expected to have different effects depending on the initial market status of the target group.

Third, if the supply of funds is perfectly elastic, small changes in $\rho^*$ can generate very large changes in credit allocation by driving groups
out of or into the market.

IV. **Modelling Federal Credit Programs**

This section defines parameters of federal credit programs and integrates government into the model described above. The government is assumed to possess the same information as banks, and may assist targeted lenders with loan guarantees, direct loans, interest subsidies or a combination of such policies.  

In a loan guarantee, the government guarantees a proportion $\gamma$ of the outstanding principal and interest, and charges a fee for this service. The proportion of default costs not covered by the guarantee fee is given by $\sigma$, where $0 \leq \sigma \leq 1$; $\sigma = 0$ signifies a fair-insurance loan guarantee fee, $\sigma = 1$ implies no fee is charged. In a pure direct lending program, the government lends an amount $S_p$ without a subsidy to the target group. In a pure interest subsidy, the government effectively agrees to pay the borrower an amount $s(<r)$ if the borrower repays the loan. Typically, direct lending will consist of a pure interest subsidy and a pure direct loan.

These credit subsidies introduce two subtle changes into the equations presented above. First, loan demand and the probability of repayment now depend on what borrowers pay, $r-s$, rather than simply $r$. Second, the return to the bank for target lending is now given by

$$\rho_T = \phi_T(r_T-s)r_T + (1-\phi_T(r_T-s))\sigma \gamma r_T - \rho_T(r_T,s,\gamma,\sigma).$$

The first term in the middle expression represents the probability of repayment times the amount due. The second term represents expected bank receipts from the government, $(1-\phi)\gamma r$, less the guarantee fee, $(1-\sigma)(1-\phi)\gamma r$. 


All credit programs are assumed to be funded by a combination of program revenues (i.e., guarantee fees and direct loan repayments) and lump-sum taxes on depositors. Thus, the financial costs of credit interventions are explicitly included. Formulae for the funding requirements of each program are derived in the Appendix.

Equilibria with credit programs parallel the equilibria presented above with minor changes. The zero profit condition for target group loans is now given by

\[
\rho^* = \rho_T (x_T, s, \gamma, \sigma),
\]

rather than (6). Sources and uses of funds are given by

\[
S(\rho^*) - X = S_G + S_T,
\]

rather than (9), where \( X \) represents government financing of credit programs. Equation (12) states that government resource requirements reduce the available supply of private credit. Equilibrium is now given by (6) -- for general borrowers only -- and (7), (8), (11), and (12). These equilibria are described formally in the Appendix.

V. Effects of Credit Subsidies

A. Market Clearing

In the market clearing equilibrium, credit subsidies have fairly standard effects. Nevertheless, it is useful to examine these results, both to show how the model works and to contrast with later results. Formal comparative statics and proofs of the Propositions and Results are provided in the Appendix.

**Proposition 1:** Unsubsidized credit policies have no effects on credit allocations or interest rates.
In providing an unsubsidized direct loan, the government simply diverts deposits from the private financial system and lends the funds directly. However, the government borrowing required to fund the program reduces private sector credit supply by the same amount as the size of the lending program. Banks respond by reducing target group loans by the exact amount of the lending programs. The net effect is simply a substitution of publicly provided target group credit for private target credit. Unsubsidized loan guarantees act as break-even insurance contracts and thus do not affect the behavior of risk-neutral banks. Thus, credit programs have real effects only if they lose money.

**Result 1:** Interest subsidies and subsidized loan guarantees operate through different channels in the market clearing equilibrium.

Subsidized loan guarantees operate through two channels. First, at any \( r_T \), guarantees raise \( \rho_T \) as defined by (10), by raising bank payments in the event of default.\(^{18}\) This effect shift effective demand vertically, and is denoted by "A" in Figure 5. Second, funding requirements reduce the supply of funds available for private lending at any level of \( \rho \).

Interest subsidies operate through three channels. First, they raise \( \rho_T \) as defined by (10), by raising the probability of repayment,\(^{19}\) thus shifting effective demand vertically. Second, subsidies require funding which reduces the available supply of funds. Therefore, the first two channels are analogous to those created by guarantees. The third effect is that subsidies drive a wedge between borrower payments and bank receipts and thereby raise target quantity demanded at any given \( \rho, r \) combination. This effect is denoted by "B" in Figure 5, and shifts target group effective demand horizontally to the right.
Effects of Loan Guarantees and Interest Subsidies on Target Group Effective Demand

Figure 5

Effects of Loan Guarantees and Interest Subsidies on the Market Clearing Equilibrium

Figure 6
Subsidized guarantees and interest subsidies raise \( \rho \), as shown in Figure 6. Through (6), \( r_G \) rises and general borrowers are crowded. Although the effects on loan payments by target borrowers are strictly ambiguous, under weak conditions, the policies reduce such payments and raise targeted loan volume. The effect on aggregate investment is also strictly ambiguous, but is positive if supply is sufficiently elastic.\(^{20}\)

All of the effects described above increase as the size of the subsidy (\( \sigma, \gamma, \) or \( s \)) increases. As the elasticity of supply rises, crowding out of general borrowers diminishes and crowding in of targeted applicants increases. Although the underlying model is decidedly different, the results in the market clearing case mirror the main insights from Penner and Silber [1973], Rao and Kaminow [1975], Plantes and Small [1981], and Silber and Black [1981].

B. Rationing and Redlining

**Proposition 2:** Interest subsidies are relatively less effective than guarantees in reallocating credit when the target group is rationed than when the target market clears.

As indicated in Section III(C) above, standard comparative statics are altered when rationing is introduced. These changes have differential effects on the impacts of guarantees and subsidies. Loan guarantees operate through the same channels as before. Interest subsidies, however, operate differently. Specifically, the third effect, the increase in quantity demanded given \( \rho \) and \( r \), has no effect on the rationed outcome, because it does not raise \( \hat{\rho}_T \) (see the dotted line in Figure 4).

The intuition is that provision of a subsidy to the borrower does little to relieve the original cause of the rationing, namely insufficient
return on lending to the target group. In particular, the direct effect of reducing borrower payments (effect "B" in Figure 5) is to shift effective demand horizontally to the right. Since, in the rationing equilibrium target group credit is determined as a residual, horizontal shifts in effective demand have no effect on the outcome. In order to induce targeted lending, the government needs instead to raise the bank return to such activity; that is, it must shift the effective demand curve vertically. Loan guarantees accomplish this task equally effectively in market clearing or rationing regimes.

Two cases are depicted in Figure 7 (where funding costs are omitted for simplicity). Initially, the target group is rationed at $E_0$ and aggregate effective demand is given by $D_0$. If the credit policy shifts effective demand to $D_1$, the target group remains rationed (at $E_1$). If effective demand shifts to $D_2$, the group's status changes to market clearing (at $E_2$). The possibility of shifting market status is increased as the elasticity of supply rises and as the change in $\hat{\rho}_T$ increases. Given that the group could have been rationed at any proportion of their notional demand, the change in status allows for the possibility of large changes in credit allocation caused by small changes in credit subsidies, if supply is highly elastic.

Therefore, in the rationing equilibrium, supply-side considerations have important implications for the effects of subsidies. If a group is redlined without credit subsidies, this reasoning follows a fortiori. Specifically, a group shifts from redlined to clearing status because of changes in bank return to lending to that group, not because of the effect of the subsidy on borrowers' demand. Such an increase is depicted in Figure 8. At the initial equilibrium, $E_0$, the target group receives no loans because $\rho^* < -0\rho_T$. Credit policy, however, can raise effective demand such
Figure 7

Credit Policies in a Rationed Equilibrium

Figure 8

Credit Policy in a Redlined Equilibrium
that \( \hat{\rho}_I > \rho^* \). In the post-intervention equilibrium, the targeted market clears.

These results stand in sharp contrast to traditional analyses of subsidies, where zero allocation would be taken as lack of willingness to pay. In imperfect credit markets, a zero allocation of credit represents an unwillingness on banks' part to lend. Therefore, subsidies, which primarily reduce borrower costs, are less effective in reallocating credit than guarantees, which directly raise the bank's return to lending, and thereby address the source of the credit constraint.

C. Interactions

As discussed in Section II above, the government simultaneously assists numerous borrower groups.

\textbf{Proposition 3:} When it subsidizes one target group, the government either partially or wholly crowds out other target groups from the market, or must raise the subsidy provided to those groups.

By subsidizing one group, the government (potentially) changes the ordering of groups by \( \hat{\rho}_I \). Because banks order groups by \( \hat{\rho}_I \) and serve them sequentially, the reordering induced by credit policy for one group can have severe effects on the other groups' credit allocation.

Suppose there are two target groups A and B. In the presence of loan guarantees for group A only, effective demand is given by \( D_0 \) in Figure 9. If \( S = S_0 \), equilibrium occurs at \( E_0 \), and all markets clear. Now suppose the government guarantees B's debt, raising \( \hat{\rho}_B \) to \( \hat{\rho}_B^1 \) from \( \hat{\rho}_B^0 \). The key point is that such a guarantee changes the relative ordering of borrower groups by \( \hat{\rho}_I \).
Figure 9

The Effects of Interactions
The new equilibrium is at $E_1$. All markets continue to clear, but some borrowing by general applicants and group A is crowded out. Thus, by subsidizing several groups, the government is to some extent offsetting the effects of its own policies. In order to maintain A's level of investment, the government must increase $\rho_A$ by raising its subsidy to group A. Therefore, credit subsidies to one group may tend to induce additional subsidies to other groups.

A more extreme case occurs if the initial equilibrium is $E_2$, representing redlining of group B. Now the guarantee to B moves the market to $E_3$ and crowds A completely out of the market, despite the willingness of the government to guarantee some portion of A's debt. Thus, in providing credit subsidies for one target group, the government can inadvertently eliminate credit for other groups it is trying to assist. In that case, the burden of credit subsidies falls on another target group. Again, in order to induce private lending to A, the government must raise the guarantee rate ($\gamma$) or the subsidy in the fee ($\sigma$). These effects are direct implications of the result that banks order borrower groups by $\rho_1$. 21

VI. Welfare

The welfare criterion employed is aggregate net wealth. 22 For simplicity, the analysis focuses on the market clearing case with infinitely elastic supply, unless otherwise stated.

Proposition 4: If borrower's riskiness (indexed by $j$) is public knowledge, all socially efficient projects and only socially efficient projects are undertaken in private equilibria.

Proposition 4 holds because if $p_I(j)$ is known for each borrower, then each borrower is essentially a different "group". Banks will set $r_I(j)$ to
satisfy $\rho = p_1(j)r_1(j)$. Since only borrowers with $R_1(j) \geq r_1(j)$, and all such borrowers choose to invest, only projects with $p_1(j)R_1(j) \geq p_1(j)r_1(j) - \rho$ are chosen, and all such projects are chosen. These, of course, represent the socially efficient projects.

**Result 2:** If borrowers' riskiness is private information, some socially efficient projects are not undertaken in private equilibria.

Specifically, the marginal borrower always has a socially efficient project in an equilibrium with no government. The marginal borrower is characterized by $R_1(j^*) = r_1$. Since the marginal borrower has a safer project than the average borrower ($p_1(j^*) > \phi_1$), it follows that

\begin{equation}
(13) \quad p_1(j^*)R_1(j^*) > \phi_1(r_1) \cdot r_1 - \rho,
\end{equation}

where the equality is just equation (6). If $pR$ is continuous in $j$, then projects by at least some $j > j^*$ will yield $p_1(j^*)R_1(j^*) > \rho$, but will not be undertaken. Therefore, in the presence of asymmetric information, the market will not, in general, fund all socially efficient projects. This inefficiency stems from the inability of banks to share in (or identify) projects with high returns (R), while they are still forced to absorb losses on defaulted projects.

**Proposition 5:** If borrowers' riskiness is private information, (small) credit subsidies financed by lump-sum taxes are welfare-improving.

Proposition 5 is the principal welfare result of the paper. Consider an interest subsidy. The costs of a program are the lump-sum taxes imposed on depositors. These taxes provide benefits to inframarginal borrowers (who would have invested without public assistance) and marginal borrowers. It can be shown that for each group, the benefits are larger than the costs to
depositors of funding subsidies for that group.

Result 2 indicates that the transfer to marginal borrowers is welfare-improving. That is, depositors could have earned $\rho$, but the expected social return to investment by marginal borrowers is greater than $\rho$. Since transfers to marginal borrowers generate expected output greater than their opportunity cost, they are welfare-improving.

The lump sum transfers from depositors to inframarginal borrowers may appear to be a straight transfer, with no welfare consequences. If that were so, Proposition 5 would have been proven. In fact, a stronger result can be derived: the lump-sum transfers that benefit inframarginal borrowers are also welfare-improving. That is, inframarginal borrowers benefit by more than depositors are made worse off. This result derives from the fact that the subsidy raises the overall group probability of repayment. Therefore, with $\rho$ constant, the rate charged by banks, $r_T$, falls. This implies that the rate paid by inframarginal borrowers, $r_T-s$, falls by more than $s$. Since the cost to the depositor of providing the subsidy is $s$, there is a welfare gain here as well.

Proposition 5 therefore implies that small subsidies can raise welfare when adverse selection exists. Proposition 4 shows that these welfare-improving capabilities occur only because of asymmetric information. Thus, the presence of asymmetric information fundamentally alters the welfare properties of the model.

A more general analysis of welfare effects in adverse selection economies is contained in Greenwald and Stiglitz [1986]. They show that in markets with adverse selection, any intervention that raises the average quality of the commodity is beneficial. Here, the concern is loan quality. Since subsidies crowd in safer projects, they reduce the probability of
default and raise average loan quality.

Proposition 6: It may also be efficient for the government to subsidize an otherwise redlined group.

This result holds because banks are unable to identify or share in projects with abnormally high returns, but are forced to absorb the costs of default. More generally, bank payoffs are a concave function of \( R \), while borrower payoffs are convex in \( R \). Thus, it is possible for the expected total return (\( pR \)) on a group's loans to exceed \( \rho \), while the expected bank return (\( \phi r \)) is less than \( \rho \). In that case, the group will be redlined, but its projects would nevertheless be socially efficient. An example is given in the Appendix.

VII. Collateral

Although the model presented above omits collateral, it is possible to generate similar results when collateral is included. Bester [1985] assumes collateral is "complete"; that is, that collateral covers the full value of principal and interest due. Under these circumstances, (1) banks are indifferent to whether default occurs, (2) loans serve only to provide liquidity, and (3) no rationing (or redlining) occurs. The first outcome is clearly counterfactual and potentially very misleading. The second outcome appears to be extremely inappropriate to the analysis of credit programs, because many subsidized borrowers (e.g., students, small businesses) frequently do not possess sufficient wealth to undertake investment without public assistance. Ruling out the possibility of rationing and redlining when analyzing recipients of credit subsidies also appears inappropriate. It is precisely these groups that are commonly
thought to be the more likely targets of rationing or redlining. Notably, the General Accounting Office [1983] cites low quantity or quality of collateral as the primary reason banks turn down private small business loan applications. Glassman and Struck [1982] report that more than one fifth of commercial bank lending to business is completely unsecured by collateral, including a sizable fraction of credit extended to small businesses.

For these reasons, incomplete collateral appears to be the appropriate assumption. Models incorporating incomplete collateral include Barro [1976], Wette [1983], Stiglitz and Weiss [1985], Bernanke and Gertler [1986], Besanko and Thakor [1987], and Calomiris and Hubbard [1988]. In such models, it is possible for collateral to play an important role without generating the extreme and inappropriate results of complete collateral. The remainder of this section shows that the basic adverse selection results derived above hold in only slightly modified form when one form of incomplete collateral is introduced.

Since banks can already identify groups, attention should focus on a single group. In the absence of collateral, if \( j > j^* \), \( \Pi(j) < 0 \). If \( j < j^* \), \( \Pi(j) > 0 \). The basic adverse selection result in this context is that all relatively riskier borrowers invest, while all relatively safer borrowers do not.

Now suppose incomplete collateral is introduced. If a bank offers contracts stipulating \((r, c)\), borrower \( j \) faces expected profits of

\[
(14) \quad \Pi(j) = p(j)(R(j)-r) - (1-p(j))c,
\]

where \( c < r \). The derivative of expected profits with respect to borrower riskiness is given by

\[
(15) \quad \frac{\partial \Pi}{\partial j} = \frac{\partial p}{\partial j} (R-r+c) + \frac{\partial R}{\partial j} p.
\]
Suppose that all projects within the group generate the same expected return; that is,

\[(16) \quad p(j)R(j) \text{ is a constant, for all } j \text{ in a group.}\]

This assumption allows the analysis to focus on the differing riskiness of alternative projects and is used by Stiglitz and Weiss [1981]. By taking derivatives of (16), it is easy to show that (15) is negative. That is, expected profit falls as \( j \) rises. Define \( j^{**} \) such that expected profits defined by (14) equal zero. Then, as in the case without collateral, all \( j < j^{**} \) do invest and all \( j > j^{**} \) do not. Therefore, the basic adverse selection result is maintained.

If (16) does not hold, a slightly modified result occurs. From (15) it is easy to see that a sufficient condition for \( \delta E \Pi / \delta j < 0 \) is

\[(17) \quad R(j) \leq r - c.\]

From (14), expected profits are negative if

\[(18) \quad R(j) = r - c.\]

Therefore, expected profits are negative for any borrower satisfying (17). Thus, once again, the safer borrowers do not invest. This situation is slightly different than the previous ones because it is not possible to show that all borrowers riskier than a certain threshold do invest. Nevertheless, because safer borrowers do not invest, adverse selection occurs at the margin.

Thus, when collateral serves primarily to reduce bank losses on defaults, the results above are unchanged. An important further research topic would be the effects of federal credit in environments where collateral serves an incentive or signalling effect.  \[23\]
VIII. Conclusion

Federal credit interventions are marked by three notable features: (1) programs employ different instruments; (2) programs target many groups simultaneously; and (3) the target groups are often thought to be rationed or redlined in the absence of credit assistance. These features generate several important results.

First, in the presence of rationing, pure interest subsidies do little to assist targeted borrowers. This occurs because the interest subsidy operates primarily by reducing borrower payments; however rationing exists because of insufficient creditworthiness of borrowers, not their unwillingness to pay. In contrast, loan guarantees operate equally effectively in either clearing or rationed regimes, because they operate primarily by raising the bank's return. These results indicate that guarantees are relatively more effective than subsidies in reallocating credit if the target group is rationed or redlined than when the target market clears. These results provide a possible basis for empirical tests for the existence of rationing.

Second, interactions among programs create perverse effects. If targeted borrowers tend to be the marginal groups, then subsidies to one group will partially crowd out or eliminate other target groups. Thus, to at least some extent, the government is simply rearranging credit among target groups and incurring program costs. These observations can help explain the rapid rise in both the volume and applications of federal lending over the past twenty years. As each group obtained credit assistance, marginal groups were increasingly crowded out and increased their subsidy requests in order to maintain their original credit level. In other words, credit subsidies create demand for more credit subsidies.
Third, because of the divergence between public and private returns, credit markets with asymmetric information typically will fund neither all socially efficient projects nor only socially efficient projects. Therefore, there is a potential welfare role for government. Small credit subsidies can sometimes improve on the market outcome. It should be stressed that the welfare results derived are generic to adverse selection models, and apply only to small subsidies. There is no basis for a conclusion that current credit policies are welfare-improving. Nevertheless, because the welfare effects are so sensitive to informational assumptions, they merit further study.

Although these results have been developed in the context of a specific model, the intuition described above should prove robust to alternative formulations. For example, the model may be extended to include collateral or equity finance. Allowing for risk-aversion by lenders dampens some of the effects, but leaves the main results unchanged (see Penner and Silber [1973] and Friedman [1978]).

The most challenging extension would separate portfolio and investment choices. For example, borrowers might want to substitute subsidized debt for equity, or subsidized capital for labor. In such cases, the change in credit will not be fully reflected in a change in real activity. On the other hand, if government provides the marginal source of funding, the change in federal credit will underestimate the change in real activity. The 1:1 assumption employed in this paper is a useful benchmark, and is used by Penner and Silber [1973], Mankiw [1986], Smith and Stutzer [1988] and many others. Further research should explore this issue in more depth, from both theoretical and empirical perspectives.
APPENDIX

I. Government Financing Requirements

To issue a $1 direct loan, the government must first raise $1, so that

\[ X(S_p) = S_p. \]  

For a loan guarantee, the expected default cost is \((1-\phi)\gamma r\), the probability of default times the guarantee rate times the contingent liability. The government subsidizes a portion \(\sigma\) of this amount, so

\[ X(\sigma, \gamma) = \sigma(1-\phi)\gamma r \]

per dollar of principal guaranteed. In a pure subsidy program, the government pays \(s\) if the project succeeds. Therefore funding requirements per dollar subsidized are given by

\[ X(s) = s\phi. \]

II. Equilibrium With Government

Equilibrium is given by (6) -- for general borrowers -- (7), (8), (11) and (12). In the market clearing equilibrium, these reduce to:

\[ \rho^* = \rho_G(r^*_G) \]
\[ \rho^* = \rho_T(r^*_T, s, \gamma, \sigma), \quad \text{and} \]
\[ S(\rho^*) - X + S_p = L^D_G(r^*_G) + L^D_T(r^*_T - s), \]

with \(\rho, r_G,\) and \(r_T\) endogenous, and where \(L^D_T = S_T + S_p\). The rationing equilibrium consists of (A-4),

\[ \rho^* = \rho_T(r^*_T, s, \gamma, \sigma), \quad \text{and} \]
\[ L^*_T = S(\rho^*) - X + S_p + L^D_G(r^*_G), \]

with \(\rho, r_G,\) and \(L_T\) endogenous, and where \(L_T = S_T + S_p\). (A-8) is
written in a form to emphasize the residual nature of targeted lending.

III. **Comparative Statics**

A. **Market Clearing Equilibria**

Details of all of the following calculations are available upon request. Total differentiation of (A-4), (A-5), and (A-6) yields \( A_0x_0 = B_0 \), or

\[
\begin{bmatrix}
1 & -\frac{\partial \rho_T}{\partial r_T} & 0 \\
1 & 0 & -\frac{\partial \rho_G}{\partial r_G} \\
\frac{\partial S}{\partial \rho} & \left( \frac{\partial \rho_G}{\partial r_T} + \frac{\partial X}{\partial r_T} \right) & -\frac{\partial L_D^T}{\partial r_G}
\end{bmatrix}
\begin{bmatrix}
d\rho \\
dr_T \\
dr_G
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\partial \rho_T}{\partial \gamma} d\gamma + \frac{\partial \rho_T}{\partial \sigma} d\sigma + \frac{\partial \rho_T}{\partial s} ds \\
0 \\
\left( \frac{\partial X}{\partial S_p} - 1 \right) ds_p + \frac{\partial X}{\partial \gamma} d\gamma + \frac{\partial X}{\partial \sigma} d\sigma + \frac{\partial X}{\partial s} + \frac{\partial L_D^T}{\partial s}
\end{bmatrix}
\]

**Direct Loans**

A change in the volume of unsubsidized direct lending has no effect on credit allocation or interest rates, because, from (A-1), \( \frac{\partial X}{\partial S_p} = 1 \).

**Loan Guarantees**

To derive the effects of other policies requires inversion of \( A_0 \). In the absence of funding requirements, the determinant is always positive. In order to avoid perverse results, while including the funding, minimal elasticities of either supply or target demand are needed to ensure a positive determinant.

The effect of changes in the guarantee rate on \( \rho \) is:

\[
(A-9) \quad \frac{d\rho}{d\gamma} = \frac{1}{|A_0|} \left[ \frac{\partial \rho_G}{\partial r_G} \left( \frac{\partial \rho_T}{\partial r_T} + \frac{\partial X}{\partial r_T} \right) \frac{\partial \rho_T}{\partial \gamma} + \frac{\partial \rho_G}{\partial r_T} \frac{\partial \rho_T}{\partial \gamma} \right]
\]

where \( |A_0| \) is the determinant of \( A_0 \). Guarantees affect \( \rho \) through two channels: they raise required government borrowing and they raise \( \rho_T \).
Using (A-2), (10), and (11), it is easy to show that

\[
(A-10) \quad \frac{d\rho}{d\gamma} = \frac{1}{|A_0|} \frac{\partial \rho_G}{\partial r_G} \sigma (1-\phi) r \left( \frac{\partial l_T^D}{\partial x_T} - \frac{\partial x_T}{\partial r_T} + \frac{\partial \rho_T}{\partial r_T} L_T^D \right) \geq 0,
\]

with equality holding only if \( \sigma = 0 \), or \( \phi = 1 \). The effect of the guarantee rate on \( r_G \) is given by:

\[
(A-11) \quad \frac{dr_G}{d\gamma} = \frac{d\rho}{d\gamma} / \frac{\partial \rho_G}{\partial r_G} \geq 0.
\]

The effect of \( \gamma \) on \( r_T \) is given by

\[
(A-12) \quad \frac{dr_T}{d\gamma} = \frac{1}{|A_0|} \left( \frac{\partial l_T^D}{\partial r_G} - \frac{\partial \rho_G}{\partial r_G} \frac{\partial r_T}{\partial r_G} \frac{\partial \rho_T}{\partial r_T} + \frac{\partial \rho_G}{\partial r_G} \frac{\partial x_T}{\partial r_T} \right).
\]

Using (A-2), (10), and (11), (A-12) reduces to:

\[
(A-13) \quad \frac{dr_T}{d\gamma} = \frac{1}{|A_0|} \sigma (1-\phi) r \left( \frac{\partial l_T^D}{\partial x_T} + \frac{\partial \rho_G}{\partial r_G} \left( L_T^D - \frac{\partial S}{\partial S} \right) \right).
\]

A sufficient but not necessary condition for the numerator to be negative (and for the determinant to be positive) is that

\[
(A-14) \quad \frac{\partial S}{\partial \rho} \frac{\rho}{S} > \frac{L_T^D}{S} \rho.
\]

Since \( L_T^D / S \) is typically much less than 1, (A-14) requires only weak assumptions. The effects of changing \( \sigma \), holding \( \gamma \) constant, are found by exchanging \( \sigma \) and \( \gamma \) in equations (A-9) to (A-13).

**Interest Subsidies**

The effect on \( \rho \) of changing \( s \) is

\[
(A-15) \quad \frac{d\rho}{ds} = \frac{1}{|A_0|} \left( \frac{\partial \rho_G}{\partial r_G} \left( \frac{\partial l_T^D}{\partial x_T} + \frac{\partial x_T}{\partial r_T} \right) + \frac{\partial \rho_G}{\partial r_G} \frac{\partial \rho_T}{\partial r_T} \left( \frac{\partial x_T}{\partial s} + \frac{\partial l_T^D}{\partial s} \right) \right).
\]
Subsidies operate through three channels. Like guarantees, subsidies raise government borrowing requirements and \( \rho_T \). Unlike guarantees, subsidies raise demand directly by driving a wedge between the interest rate borrowers pay and the rate lenders charge. All three effects force \( \rho \) upward. As a consequence, \( r_G \) rises as well, and general borrowers are crowded out.

The effect of subsidies on \( r_T \) is given by

\[
\frac{dr_T}{ds} = \frac{1}{|A_0|} \left\{ \left( \frac{\partial L^D}{\partial r_G} - \frac{\partial \rho_G}{\partial r_G} \frac{\partial s}{\partial r} \right) \frac{\partial \rho_T}{\partial s} + \frac{\partial \rho_G}{\partial s} \left( \frac{\partial x}{\partial s} + \frac{\partial L^D}{\partial s} \right) \right\}
\]

Although the sign of (A-16) is ambiguous, it is also less interesting than whether \( dr_T/\partial s \leq 1 \), since borrowers pay an effective rate of \( r \)-s. It can be shown that a sufficient condition for \( dr_T/\partial s < 1 \) is that

\[
\frac{\partial s}{\partial \rho} \frac{\rho}{S} > \frac{\rho}{S}
\]

B. Rationed Equilibria

Total differentiation of (A-4), (A-7), and (A-8) yields \( A_1x_1 = B_1 \), or

\[
\begin{bmatrix}
1 & 0 & 0 \\
1 & 0 & -\frac{\partial \rho_G}{\partial s_T} \\
\frac{\partial s}{\partial \rho} \left( 1 + \frac{\partial x}{\partial s_T} \right) & -\frac{\partial L^D}{\partial s_T}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \rho_T}{\partial \gamma} \\
\frac{\partial \rho_T}{\partial \sigma} \\
\frac{\partial \rho_T}{\partial s}
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial \rho_T}{\partial \gamma} \ d\gamma + \frac{\partial \rho_T}{\partial \sigma} \ d\sigma + \frac{\partial \rho_T}{\partial s} \ ds \\
0 \\
\left( \frac{\partial x}{\partial s} p - 1 \right) ds_p + \frac{\partial x}{\partial \gamma} \ d\gamma + \frac{\partial x}{\partial \sigma} \ d\sigma + \frac{\partial x}{\partial s} \ ds
\end{bmatrix}
\]

Direct Loans

As before, unsubsidized direct loans are neutral, since the coefficient on \( ds_p \) is zero.

Loan Guarantees

Because the policy parameters completely determine \( \tilde{r}_T \), the analysis of policy is simplified considerably. Inversion of \( A_1 \) shows that
\[ |A_1| < 0. \] The effect of \( \gamma \) on \( \rho \) is given by

\[(A-18) \quad \frac{d\rho}{d\gamma} = \frac{\partial \rho_T}{\partial \gamma} \geq 0.\]

That is, the general equilibrium effect of a shift in \( \gamma \) is captured fully by the rise in the \( \rho_T \) curve.

The effect of guarantees on target group loan allocations is given by:

\[(A-19) \quad \frac{dS_T}{d\gamma} = \frac{1}{|A_1|} \left\{ \left[ \frac{\partial L^D}{\partial r_G} \frac{\partial \rho_T}{\partial s} + \frac{\partial \rho_G}{\partial r_G} \frac{\partial X}{\partial s} \right] \right\} \]

\[(A-19) \] is positive under precisely the same conditions for which \( \frac{d\gamma}{d\gamma} < 0 \) in the market clearing case (see (A-12)). Thus, guarantees work in exactly the same ways in clearing and rationed equilibria.

**Interest Subsidies**

The effect on \( \rho \) is given by

\[(A-20) \quad \frac{d\rho}{ds} = \frac{\partial \rho_T}{\partial s}.\]

The effect of subsidies on targeted loans is given by

\[(A-21) \quad \frac{dS_T}{ds} = \frac{1}{|A_1|} \left\{ \left[ \frac{\partial L^D}{\partial r_G} \frac{\partial \rho_T}{\partial s} + \frac{\partial \rho_G}{\partial r_G} \frac{\partial X}{\partial s} \right] \right\}. \]

The numerator is the same as (A-16), except that the direct effect of the subsidy on loan demand, \( \frac{\partial L_T^D}{\partial s} \), does not appear in the rationed equilibrium.

**IV. Proofs of the Propositions and Results**

**Proposition 1:** For direct loans, substitution of (A-1) into (A-6) or (A-8) implies neutrality. For guarantees, substitution of \( \sigma = 0 \) in (A-2), (A-10), and (A-13) implies neutrality.
Result 1: In (A-9) and (A-12), $\gamma$ directly affects $\rho_T$ and $X$. In (A-15) and (A-16) $s$ directly affects $\rho_T$, $X$, and $l^D_T$.

Proposition 2: Compare the ratios (A-12)/(A-19) and (A-16)/(A-21).

Propositions 3, 4: Result 2: See the text.

Proposition 5: Consider an interest subsidy. (Similar calculations for loan guarantees are available upon request.) Let $j_0$ ($j_1$) be the marginal borrower before (after) the subsidy is imposed, and $b_0$ ($b_1$) be borrower payments before (after) the subsidy is imposed. That is $b_0 = r^0_T$, where $r^0_T$ is the target rate determined in a private equilibrium; $b_1 = r^1_T - s$, where $r^1_T$ is determined in an equilibrium with government. Since $\rho$ is fixed,

\begin{equation}
(A-22) \quad r^0_T > r^1_T.
\end{equation}

The benefit to inframarginal borrowers is $\int_0^j p_j (b_0 - b_1) f(j) dj$. That is, inframarginal borrowers benefit from the subsidy only if their project succeeds and they benefit in that case by $b_0 - b_1$. The benefit to marginal borrowers is $\int_0^1 p_j (R_j - b_1) f(j) dj$. They benefit only if their project succeeds, and each successful borrower is better off by $(R_j - b_1)$. The direct cost to depositors is $\int_0^1 p_j f(j) sdj$, which can be obtained from (A-1), (3), and (5). The bank's net earnings on marginal investments is $\int_0^1 [p_j (b_1 + s) - \rho] dj$. These earnings are distributed to depositors. Subtracting costs from benefits yields a welfare change of

\begin{equation}
(A-23) \quad dw = \int_0^j p_j (b_0 - b_1 - s) f(j) dj + \int_0^1 (p_j [(R_j - b_1) + (b_1 + s) - s] - \rho) f(j) dj.
\end{equation}
The first term on the right side is positive because of (A-22). The second term reduces to \( \int_{j_0}^{j_1} (p_j R_j - \rho) f(j) dj \). If \( j_1 \) is sufficiently close to \( j_0 \), this term is also unambiguously positive due to Result 2. Therefore for small \( s, \ dw > 0. \)

**Proposition 8:** Let \( p(j) = j \) and \( f(j) \) be uniform. Then \( \phi(r(j*)) = j*/2 \). Let \( R(j) = 3 - j \). Then \( p(j)R(j) = 3j - j^2 \). The total return to the groups' projects is given by

\[
(A-24) \quad \int_{0}^{1} (3j - j^2) dj = 7/6.
\]

The bank return to lending to this group at rate \( r \) is given by

\[ \rho_T = \phi(r) \cdot r. \]  
Any interest rate \( r* \) can be expressed as \( r* = 3 - j* \). Banks offering \( r* \) earn \( (3-j*)j*/2 \). As \( j \) increases, this return increases, but at \( j* = 1, \rho_T = 1 \). Thus, the bank return to targeted lending is never greater than 1. If \( 1 < \rho* < 7/6 \), then target group investment is socially beneficial, but will not be undertaken in private equilibrium.
FOOTNOTES

1 Several recent related papers should be mentioned here. Smith [1983] discusses optimal government lending with imperfect information, but his analysis focuses on monetary policy rather than selective credit subsidies. Chaney and Thakor [1985] analyze the effects of emergency loan guarantees. However, as they point out, these guarantees should be distinguished from standard ongoing loan guarantee programs, which guarantee many loans, smaller amounts per loan, and do so at the time the loan is made. Bosworth, Carron, and Rhynne [1987] present a large amount of information and discussion concerning all aspects of federal credit. They do not, however, present any formal analysis.

2 Detailed information concerning federal credit programs may be found in "Special Analysis F," Special Analyses: Budget of the U.S. Government, any year, or Bosworth, Carron, and Rhynne [1987]. Unless otherwise cited, data presented in this section are obtained from "Special Analysis F," Special Analyses: Budget of the U.S. Government, selected years. For full citations, see Gale [1987], chapter 2.

3 One reason for the popularity of loan guarantees is that the budget records a guarantee as a zero outlay. See the Congressional Budget Office [1984] for further discussion.

4 The five Government-Sponsored Enterprises are Federal Home Loan Banks, the Federal National Mortgage Corporation, the Federal Home Loan Mortgage Corporation, the Student Loan Marketing System, and the Farm Credit System (FCS). Only the FCS operates in primary markets.

5 The model is formally a one-period model, but can be rewritten in an explicit two-period format without changing any of the results.

6 More generally, the results hold if projects are larger than borrower net worth. See Calomiris and Hubbard [1988], for example.

7 Stiglitz and Weiss [1981] assume that \( p_R \) is constant across \( j \) for a given group.

8 Formally, 
\[
\frac{\partial \phi_I}{\partial \tau_I} = \frac{\partial f_I(j^*)}{\partial \tau_I} / \left( p_I(j^*) - \phi_I \right) < 0 \text{ because } p_I(j^*) > \phi_I.
\]

9 From (4), an equivalent statement is that \( \phi_G(r) > \phi_T(r) \) for all \( r \).
One set of sufficient conditions for either statement to hold are \( f_T = f_G \) and \( p_T(j) < p_G(j) \).

10 Throughout this paper, I follow the Stiglitz-Weiss (1981, pp. 394-5) definitions of credit rationing and redlining: rationing exists if "among loan applicants who appear to be identical some receive a loan and others do
not and rejected applicants would not receive a loan even if they offered to pay a higher interest rate;" redlining exists if "there are identifiable groups or individuals in the population who, with a given supply of credit, are unable to obtain credit, even though with a large supply of credit they would" receive loans.

11. Riley [1987] provides an independent derivation of this result.

12. A supply curve that intersected aggregate effective demand on the horizontal section to the left of $S_3$ would entail rationing of general borrowers and redlining of the target group. This equilibrium is ignored because it is similar to the $S_3$ equilibrium.

13. Although this explanation is couched in competitive terms, it is easy to show that more subtle equilibrium concepts, such as Riley's [1979] reactive equilibrium also imply a "general borrowers first" rule. Wilson's [1977] anticipatory equilibrium also implies the same rule, provided that banks can add contracts as well as drop them in response to a defection.

14. There is an infinitesimal probability that groups with $\rho_T = \rho^*$ have clearing or redlined credit markets, depending on the position of the supply curve in Figure 4.

15. Government could also attempt to assist certain groups by passing usury laws, setting portfolio restrictions, requiring banks to lend at some rate to any borrower, or other policies. See Ordover and Weiss [1981], and Penner and Silber [1973], for example.

16. For simplicity, the fee is assumed to be paid by the bank. However, the incidence of the fee is independent of who actually pays the government. See Gale [1988], Appendix A.

17. For simplicity, the lump sum tax is assumed to be financed wholly from savings. This guarantees that the costs of credit programs are recorded and thus will serve to assure that welfare changes are not brought about simply by shifting resources from some other (unmodeled) sector into the credit market. Other specifications, in which only a portion of the tax is financed by reduced saving, yield similar qualitative results.

18. A full (100%) guarantee creates a horizontal $\rho_T$ curve. Any partial guarantee raises, but retains the original shape of, the $\rho_T$ curve given in Figure 2.

19. Gale [1988] has estimated that, for supply elasticities exceeding 0.5, aggregate investment rises.

20. Because $\phi_T = \phi_T(r_T - s)$, $\frac{\partial \phi_T}{\partial s} = -\frac{\partial \phi_T}{\partial r_T} > 0$.

21. Some of the insights in this section have been recognized independently by Bosworth, et al. [1987]. They do not provide any formal analysis, however. Penner and Silber [1973] focus on interactions among
programs that target the same group.

22 The same criterion is employed by Mankiw [1986]. For examples of Pareto-improving credit policies in different models, see Smith and Stutzer [1988] and Gale [1987], Appendix C.

23 For preliminary research along these lines, see Gale [1987], Appendix C, or Gale [in progress].
REFERENCES


__________ [in progress], "Collateral, Incentives, and Default in Federal Credit Programs".


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