INTERTEMPORAL PRICING IN SEARCH MARKETS.

CUSTOMER MARKETS AND PRICE RIGIDITY

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December 1992

UCLA
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Working Paper #681

Gary Becker, Roland Benabou, Andrew Dick, Nancy Stokey, and specially Anne Villamil provided valuable suggestions. This is an extremely preliminary version. The many errors here are my own responsibility.
ABSTRACT

I provide an equilibrium search model, in which consumers search in order to find low price sellers, and sellers set prices in order to maximize the value of the firm. Both sellers and buyers are infinitely lived, and customer relationships evolve. In this way, I provide a microfoundation to the idea of "customer markets".

Price rigidity obtains from kinked demand curves. These kinks arise from an asymmetry at the extensive margin: downward adjustments could only attract new customers, while upward adjustments discourage not only newcomers, but also "old" customers.

Lower intertemporal correlation of prices increases market power in a market where information is costly. It also induces a less efficient size distribution of sales (high cost firms produce a larger proportion of industry output in more unstable markets.)
1. INTRODUCTION

This paper investigates behavior (in particular, consumer search patterns and firm pricing strategies) in markets characterized by repeat purchase. The class of goods transacted in such markets includes non-durables, personal services, and intermediate goods regularly purchased by user firms.

This work formalizes in an explicit search-equilibrium framework, some ideas in Industrial Organization and in Macroeconomics that go back to Stigler (1961) and to the volume *The Microeconomic Foundations of Employment and Inflation Theory* (Phelps et al. 1970).

I prove Stigler's conjecture about the dependence of search patterns on the intertemporal correlation of prices. The higher the correlation, the more search intensity is tilted towards the initial period. This describes very accurately the common practice of new buyers of shopping around for a while until an acceptable supplier is found. A client relationship is established for as long as the price (and other relevant attributes) remains within bounds (or until new information about better trading opportunities arrives "exogenously").

The effort to reconcile microeconomic theory with some macroeconomic phenomena such as unemployment, price rigidities and short run effects of monetary policy had a high point in the late 60's and early 70's, with the work of Robert Lucas, Edmund Phelps and others. There is a current revival of that attempt. (See for instance Blanchard and Fischer 1989 and Mankiw and Romer 1991.) The current emphasis is evenly divided between product and
labor markets. This paper is concerned with product markets. These markets have been analyzed with different "representative market structures." Lucas (1972 and 1973) worked within the price taking competitive paradigm. Rotemberg and Saloner (1986) use a game-theoretic oligopoly approach. This paper is in the "atomistic competition" line of Phelps and Winter (1970). Here, informational imperfections generate downward sloping demands at the individual firm level for a homogeneous good. I provide a search theoretic foundation to the idea of "customer markets", developed by Okun (1981) and Phelps and Winter (1970). Fishman and Rob (1991) provide an interesting treatment of similar issues. One of their most suggestive findings is that essentially identical firms may be destined for different long run fates by historical accident (favorable initial cost shocks). Their model studies discrete, namely two, cost and price distributions and does not have the price rigidity implications I identify here. Rustichini and Villamil (1991) provide another microfoundation for price rigidity. They focus on demand shocks in a single buyer and seller model, while I focus on idiosyncratic cost shocks in a multi-agent model. However, stochastic processes with stationary distributions drive (to some extent) both sets of results. In addition, this paper provides a framework for better understanding the welfare effects of inflation, an issue that I discuss at length in Tommasi (1992).
II. THE MODEL\(^1\)

There is a continuum \([0,1]\) of consumers. Each of them has a cost of search \(c\) per visit, where \(c\) is distributed uniformly in \([0,C]\). There is a continuum of firms, indexed by \(f\in[0,1]\). Firms have unit production cost \(\theta_t^f\). This cost follows the stochastic process:

\[
\begin{align*}
\theta_{t+1}^f &= \theta_t^f \\
&= \text{a drawing from } \Psi(\theta) \\
&\quad \text{with probability } \rho \\
&= \text{a drawing from } \Psi(\theta) \\
&\quad \text{with probability } (1-\rho)
\end{align*}
\]

where \(\Psi(\theta)\) is a cumulative density function, with support \([\theta, \bar{\theta}]\). This process will induce a similar one for output prices, as will be shown later. This specification captures the dynamics of a market subject to exogenous idiosyncratic shocks in a way such that a stationary steady state distribution is maintained.\(^2\) The process (1) permits a particularly simple (and illuminating) comparative statics on the parameter \(\rho\). Lower \(\rho\)'s may represent industries that are undergoing rapid changes (the computer industry?), or economies where relative prices are more volatile (high inflation countries?). Here I abstract from the level effects (say all prices moving together on average, related to variability of relative prices across different industries), and concentrate on relative prices across sellers (variability within the industry).

\(^1\)I am indebted to Roland Benabou for bringing to my attention his 1988 paper (Benabou 1988), which deals with related issues. There, he develops a general model for the case of time invariant costs (hence prices).

\(^2\)In the case where the parameters of the model are such that firms with high cost are unable to sell, (and if we interpret output equal zero as exit) this dynamics includes entry and exit decision. The specification could capture, in a very stylized way some characteristics of industry evolution (Hopenhayn 1990): high variance of growth rates across firms, high dispersion in size, and significant turnover.
The next section studies the consumer problem. The main implication is that information stocks are decreasing (acceptance prices increasing) in price instability. Section B studies the pricing problem of firms, taking into account customer loyalty, what generates intertemporal complementarities (demand at time t is negatively related to price at time t-j). Section C looks at the equilibrium, and Section D provides the comparative statics.

A. The Consumer Problem

Each consumer purchases one unit per period and tries to minimize the expected present value of total expenditure, including search costs. We could easily extend to more general intertemporal preferences (see Benabou 1988), but I restrict to this simple case to focus on the search-induced aspects of the problem.

The stochastic process for individual firm prices is:

\[ p_{t+1}^f = p_t^f \quad \text{with probability } \hat{\rho} \]
\[ = \text{a new drawing from } F(p) \quad \text{with probability } (1-\hat{\rho}) \]

The distribution \( F(p) \), as well as the stochastic process (2), will be derived from firms' optimizing behavior in equilibrium. Notice the use of \( \hat{\rho} \) rather than \( \rho \). I show later that the probability of maintaining price (\( \hat{\rho} \)) is higher than that of a new cost shock (\( \rho \)). This "stickiness" is generated by a Sweezy type demand curve, as discussed in the next section.

It is shown in Tommasi (1992) that the optimal solution to this problem implies:

1) Accepting any quotation below \( R \) and rejecting any quotation above it, where \( R \) is a reservation price which depends on the cost of search \( c \).

2) Each period's search starts by coming back to the store where last
period's unit was purchased.\(^3\)

The reservation price is defined implicitly by:

\[
(3) \quad c = \frac{1}{1-\rho}\int_0^R (R-x) \, dF(x)
\]

The marginal cost of an additional search is equated to the marginal benefit: the expected price reduction converted into an expected present value through the discount factor \(\beta\) and the probability \(\hat{\rho}\).

The distribution of consumer search costs is mapped into a distribution of reservation prices. Comparative statics from (3) shows reservation prices decreasing in \(\hat{\rho}\): the more stable the environment, the more informed consumers will be and the lower the prices they will accept. In equilibrium, this will induce mark-ups decreasing in consumer information. In the extreme case in which \(\rho=\beta=1\), the benefit of search is infinite (analogous to cost of search zero for all buyers) and customers insure the lowest possible price for themselves. In such a scenario, all sales will go to firms with the lowest marginal cost \(\theta\), and they will charge a price equal to that marginal cost, the competitive (Bertrand) result. As long as \(\rho<1\) (or \(\beta<1\)), given positive (and heterogeneous) search costs, firms will face downward sloping demands and will have price-setting power. This monopoly power is decreasing in \(\rho\).

The expected number of stores visited in the first period \((n_1)\) and in steady state \((n_t)\) equal:

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\(^3\)This result is obtained under two assumptions. A) The cost of recall is a (small) negative number (say you enjoy buying again from the same supplier). This insures the existence of a unique reservation price (see Tommasi 1990 for a model with both a "recall" and a "purchase" reservation price). B) Consumers believe that if the price changes, it will be a new drawing from the distribution \(F\). This allows a reservation price rule independent of the common history of the firm-customer pair.
\[ n_1 = \frac{1}{F(R)} \quad \text{and} \quad n_t = \hat{\rho} + (1 - \hat{\rho}) \left[ \frac{1}{F(R)} \right] \]

The extreme cases of $\hat{\rho} = 0$ and $\hat{\rho} = 1$ correspond, respectively, to the static case ($n_t = 1/F$, as in Sargent 1987, Chapter 2) and to the case of time-invariant prices. This latter case implies $n_t = 1$, which is exactly Stigler's (1961, p.218) conjecture: "If the correlation of asking prices of dealers in successive time periods is perfect, the initial search is the only one that need be undertaken".

B. Demand and Firm Optimization

Each firm $f$ has as objective the maximization of the expected present value of the profit stream. Per period profits are $(p_t - \theta_t)q_t$, where $q_t$ is the quantity sold at time $t$. Given the intertemporal links described in the consumer problem, which we could call "goodwill", sales at $t$ will be dependent upon past prices. Let $\phi_t^f$ be the customer base that firm $f$ has at the end of period $t$. This $\phi$, which will be our state variable for the dynamic problem of the firm, is a function that maps the set of consumer search costs $[0,C]$ into the nonnegative real numbers; $\phi_t^f(c)$ is the number of consumers of type $c$ that purchased from firm $f$ at time $t$.

The dynamic programming problem for the firm is, then:

\[ V(\phi_{t-1}, \theta_t) = \max_p (p - \theta_t)q(\phi_{t-1}, p) + \beta E[V(\phi_t, \theta_{t+1}) | \theta_t] \]

Let $\phi_N^f(c)$ be the number of buyers of type $c$ that arrive every period to any particular firm ($N$ stands for New customer), which equals the probability that a customer of type $c$ will be involved in search. A buyer will be searching if the firm from which he was buying changed its price, and the new
price is above his reservation level, so that \( \phi_N(c) = (1 - \hat{\rho})[1 - F(R(c))] \). Let \( P_\phi \)
be defined by:

\[
(5) \quad P_\phi(c) = \begin{cases} \phi(c) & \text{for } c \geq c(p) \\ 0 & \text{for } c < c(p) \end{cases}
\]

where \( c(p) \) is the cost of search of consumers with reservation price \( p \), as
defined in (3). That is, \( P_{\phi_{t-1}}(c) \) is the number of consumers of type \( c \) that
reanvassed the firm at time \( t \) and found its price \( p \) to be acceptable, and
hence will also show up at \( t+1 \). Similarly, \( P_{\phi_N}(c) \) will be the number of type
\( c \) consumers that come to the firm for the first time and find the price \( p \)
acceptable. Now we can write the law of motion for the state as:

\[
(6) \quad \phi_t = P_{\phi_{t-1}} + P_{\phi_N}
\]

**Proposition:**

There exists a unique solution to the dynamic programming problem (4) and
(6). The policy function \( p(\phi, \theta) \) is defined implicitly by:

\[
(7) \quad FMR(p^-) \leq \theta \leq FMR(p^+)
\]

where

\[
(8) \quad FMR(p) = \frac{q(\phi, p)}{\partial q(\phi, p)/\partial p} + p + \beta \frac{\partial EV(\phi, p)/\partial p}{\partial q(\phi, p)/\partial p}
\]

is defined only at points where those derivatives exist, and \( FMR(p^-) \)
\( (FMR(p^+)) \) refers to the limit from the left (right).

**Proof:** in the Appendix.

Notice that the first two terms in (8) conform the standard (myopic)
marginal revenue as a function of price. Full marginal revenue \( (FMR) \)
includes the effect of current price on the expected value of the future
stream of profits. The last term being negative (higher price today implies
a smaller pool of repeat buyers in the future), the price that solves (7) will be smaller than the one that maximizes per-period profits. Repeat business encourages lower prices because of complementarity between sales in different periods due to these "customer relationships" (goodwill). The pair of inequalities in (7) replaces the standard first order condition because the return function is non differentiable at points where demand q(p) has kinks. The return function has kinks because customers discontinuously leave the firm and shop elsewhere after one bad price. This occurs because buyers know that if they have gotten one bad price, they are likely to get a string of bad prices because of the persistence in the cost process. The kinks happen at prices which were absolute maxima going backwards in time. Each of those prices drove away customers with low R, introducing truncation in ϕ, as specified in (6). The last price charged belongs to that set, i.e. there is a kink at \( p^f_{t-1} \) and this Sweezy type rule obtains.\(^4\) This explains why \( \hat{\rho} > \rho \).

C. Equilibrium

I will assume that the distribution of search and of unit production costs is such that no firm will ever have a cost higher than the maximum acceptable price. If that were not the case, some potential firms will not make positive sales. The main additional insight that case (analyzed in Tommasi 1992) provides is the fact that in more unstable (lower \( \rho \)) environments, firms with higher costs might be in operation. This is consistent with the intuition that high inflation permits the survival of

\(^4\)The kink is caused by customer-seller interaction and not by seller-seller interaction, which is the standard case (Sweezy 1939).
some firms that wouldn't be able to survive in a less noisy environment. As we shall see, such finding is just a corner case of a more general result: that lower intertemporal correlation of costs (hence of prices) generates tighter size distribution of firms, due to the reduced "arbitrage" incentives that buyers have.\(^5\) This implies that high cost firms produce a larger proportion of output in more unstable markets; generalizing to the corner, some firms produce at all just because of diminished buyer information.

An equilibrium will be a stationary distribution of prices \(F(p)\) such that: 1) consumers follow the reservation price rule (3) and start each period by recanvassing the last seller, and 2) firms set prices according to (7)-(8). This \(F(p)\) is the distribution from where customers believe they are drawing quotations (as specified in (2)) and from where sellers believe their competitors' prices are random drawings. Given the existence of a continuum of buyers and of a continuum of sellers, by the law of large numbers, this expected distribution will coincide with the actual one in equilibrium. This distribution is obtained by applying the change of variable formula to the pricing function \(p(\phi, \theta)\):

\[
(9) \quad f(p) = \frac{\partial \theta}{\partial p} \psi(\theta(p))
\]

\[
= 0 \quad \text{for } p \in [m, M]
\]

\[
= 0 \quad \text{elsewhere}
\]

where \(m = p(\bar{\theta}), M = p(\bar{\theta}), \psi\) is the density of unit cost \(\theta\), and \(\partial \theta/\partial p\) is the derivative of the unconditional (independent of price history) mapping from \(p\) to \(\theta\). Existence and conditions for uniqueness are shown in the Appendix.

\(^{5}\)Blejer and Hillman (1982) use a similar intuition, in the context of exchange rates.
D. Comparative Statics

The effects of price instability are studied by performing comparative statics on the shock parameter \( \rho \); the lower it is, the more instability. The results here are analogous to the ones obtained in Tommasi (1992), where firms are assumed to behave myopically (maximizing per-period profits). In that paper the results were due to the diminished consumer information. Here that is reinforced by reducing the intertemporal link in the firm pricing problem (in the extreme of \( \rho=0 \), no complementarities would exist). Let me summarize the main results, which are shown in the Appendix.

There is an increase in markups, that is for any given unit cost, price charged is higher. This is due to an "upward shift" in the demand curve faced by the individual firm. This shift, in turn, reflects higher reservation prices, as shown in Section A. This increase in prices is, obviously, welfare decreasing for consumers.

Furthermore, there is a change in the composition of the industry. Geometrically, this is reflected in a steeper demand curve: a bigger fraction of total output will be produced by firms in the range of high prices (high unit cost). This has the effect of reducing efficiency, by increasing industry total cost of producing any given rate of output. The matching between customers and lower cost stores gets disturbed by the instability in the system.

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\[6\text{Remember that this is a comparative statics exercise. I am not studying dynamic adjustments to a change in } \rho.\]
III. CONCLUSION

We have described a search market in which customer relationships develop to economize on information acquisition. The stochastic nature of the cost process induces price variability, generating flows of customers towards lower price places. Price instability (which in Tommasi 1992 is related to inflation) generates inefficiencies. More generally, the presence of noise makes the system less effective in dealing with fundamentals.

This can be extended in several directions. On the Industrial Organization side, a more general stochastic process for cost can be allowed. If firm's have a permanent idiosyncratic component (what seems to be true in most actual markets), there will be a learning component to the search activity. In this line we could also study entry: Which is the optimal pricing strategy for a new entrant? This issue has been studied in the marketing literature (see for instance Winer 1986 and Dwyer et al. 1987) and in Bagwell (1985) for the case of a monopolist. Once again, allowing for imperfect information (or habit formation) in attributes other than price could prove worthwhile.

On the Macroeconomics front, we found reasons for real price rigidity at the individual level. It would be interesting to introduce market-wide shocks to see whether aggregate rigidity obtains. For this purpose, the dynamics of learning will be necessary too.

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7 This model refers to price information, but could be rewritten in terms of any other attribute.
8 McMillan and Morgan (1988) answer nay in a model with stationary price paths. Their answer is conditioned by the stationarity in their model (in the absence of common shocks no one changes price). It might be the case that the mix of idiosyncratic and common elements could support price rigidity.
The set up in this paper seems to provide a natural framework for the study of those questions.

REFERENCES


