Habit Persistence in Overlapping Generations

Economies Under Pure Exchange

Amartya Lahiri
University of California, Los Angeles

Mikko Puhakka
Academy of Finland and University of Lapland

Working Paper Number 754
Department of Economics
University of California, Los Angeles
Bunche 2263
Los Angeles, CA 90095-1477
May 24, 1996
Habit Persistence in Overlapping Generations Economies

Under Pure Exchange*

by

Amartya Lahiri
University of California, Los Angeles

and

Mikko Puhakka
Academy of Finland and University of Lapland

May 1996

Abstract: This paper analyzes the implications of habit persistence preferences for savings and equilibrium dynamics in the context of an overlapping generations model under pure exchange. We show that habit persistence can convert an economy which otherwise has no role for government currency into an economy where there does exist a role for such currency. Further, the increased saving induced by habit persistence implies that governments are able to float higher levels of deficits in such economies relative to economies without habit persistence. The paper also shows that relative to standard preferences, habit persistence preferences cause the savings function to become decreasing in the rate of interest at lower values for relative risk aversion. This is of particular interest since the paper demonstrates that this result is closely related to the relative success of models with habit persistence in explaining the equity premium puzzle.

*We would like to thank, without implicating, Costas Azariadis and Philippe Weil for helpful discussions at an early stage of this paper as also seminar participants at UC Davis and University of Colorado, Boulder. This paper was written while Puhakka was visiting the Department of Economics at UCLA. He thanks the Academy of Finland for financial support.
1. Introduction

Two features of US data that have consistently embarrassed general equilibrium models based on complete markets and optimizing agents over the past decade and a half are the equity premium puzzle and the excess sensitivity of consumption. The first refers to the average excess return of 6 percentage points of the stock market over Treasury bills while the second refers to the strong correlation of consumption growth with income. Attempts at reconciling the data with the predictions of standard models of asset pricing and consumption have contributed to the emergence of a large literature over this period.

One of the approaches that researchers have taken in order to "fix" the problem has been to introduce habit persistence preferences into otherwise standard models. Deaton (1992) provides a summary of some of the work in consumption involving habit persistence. The model with habit persistence generates smoother consumption paths than the traditional models of consumption. Furthermore, using habit formation one can explain the observed correlation of the first difference in current consumption and the first difference in lagged income. In the asset pricing literature Constantinides (1990) has shown that habit persistence is able to account for the equity premium puzzle without implying an implausibly high degree of relative risk aversion on the part of households.\footnote{Other studies which use these preferences to reconcile US data on equity premia and consumption are Burnside (1994), DeTemple and Zapatero (1991), Ferson and Constantinides (1991) and Heaton (1995). Abel (1990) used an alternative but related method to reconcile the data. He introduced the 'keeping up with the Jones' preferences. This, however, proved relatively unsuccessful since in practice it required a relatively high rate of risk aversion to account for the asset pricing puzzle. For an overview of this literature see Kocherlakota (1996).} \footnote{Black (1995) strongly advocated the use of time non-separable preferences in dynamic general equilibrium models which are commonly used. One of the methods he suggested was habit persistence. However, we}
More recently, habit persistence has also been introduced into optimal growth models and the standard business cycle model. Orphanides and Zervos (1994) introduce habit persistence into the optimal growth model such that preferences over present and past consumption are jointly non-concave. They show that the model can exhibit multiple steady states. Boldrin, Christiano and Fisher (1995) introduce habit persistence in a two-sector business cycle model and demonstrate that the model is able to account for the observed persistence in output growth - a fact which standard business cycle models find hard to explain.

In this paper we examine the consequences of habit persistence in an overlapping generations model with pure exchange. We study the impact of habit persistence on the savings behavior of young agents as well as its effects on the dynamics of the economy. In particular, we compare the behavior of our model to that of a standard overlapping generations model with pure exchange (e.g., see Azariadis (1993)) and show that the models behave very differently.

We introduce habit persistence into an otherwise standard exchange economy overlapping generations model with two-period lived agents by assuming that the second period utility of the two-period lived agents depends on first period consumption. In particular, higher first period consumption reduces the utility derived from a given level of second period consumption. Our formulation of habit formation is quite simple but at the same time rather standard in the literature.

should note that Black was not a proponent of introducing habit persistence by having past consumption create a floor for current consumption which is the common way in which it is introduced in most of the literature and this paper.
It attempts to capture the detrimental effect of a stock of habits (formed through past consumption) on current utility.\textsuperscript{3}

The paper shows that, ceteris paribus, habit persistence increases the desired savings of the young. A direct implication of this is that the government is able to float higher levels of deficits in an economy with habit persistence relative to an economy without habits. We also demonstrate that habit persistence can change a "classical" (using David Gale's (1973) terminology) economy into a "Samuelsonian" economy, i.e., it transforms an economy without any role for the provision of government currency into one where there is a role for such a provision.\textsuperscript{4} Further, we show that habit persistence preferences could have dramatic effects on the stability properties of the stationary state of the model. In particular, it could convert an economy with a stable monetary steady state into an economy with an unstable monetary steady state.

It is known from Gale (1973) and Grandmont (1985) that an overlapping generations economy with pure exchange can exhibit a stable monetary steady state and cycles. In standard models this requires the savings function to be decreasing in the interest rate. As is well known, downward sloping savings functions typically arise due to very strong concavity of the utility function. The paper shows that, ceteris paribus, habit persistence preferences induce downward sloping savings functions and, hence, backward bending offer curves with relatively weaker

\textsuperscript{3} In infinite horizon models the stock of habits is typically modeled as a weighted sum of all past consumptions. However, in the model of this paper agents live for two periods and, hence, the only past consumption is first period consumption. Thus, our formulation is consistent with the typical formulation of habit persistence in the literature.

\textsuperscript{4} Azariadis and Smith (1993) show the same result in a pure exchange overlapping generations model. Their result is based on credit rationing due to adverse selection (informational asymmetries between lenders and borrowers) which increases savings.
concavity of the utility function. This result suggests that habit persistence preferences have potentially important implications for the possibility of cycles, sunspot equilibria and stable monetary steady states with positive outside money since these three are interconnected phenomena.\(^5\)

The result regarding the slope of the savings function is also connected to the success of models with habit persistence in explaining the equity premium puzzle. The asset pricing literature has concentrated on finding ways to generate high equity premia without resorting to implausibly high degrees of concavity of the utility function (as measured by the coefficient of relative risk aversion). The introduction of habit persistence achieves this task by making consumption services smaller than consumption purchases.

Following Weil (1992), it is perhaps easiest to illustrate the implication of habit persistence through a standard isoelastic felicity function of the form \(u(c) = \frac{(c - h)^{-\sigma}}{1 - \sigma}\) where \(h\) is the stock of habits. It is easy to check that \(-\frac{c u''}{u'} = \sigma \left( \frac{1}{1 - h/c} \right) > \sigma\). Thus, agents become more risk averse for a given \(\sigma\). However, our results show that this is also the reason behind the downward sloping savings functions and backward bending offer curves that now emerge at smaller values for \(\sigma\). The close connection between our result and the literature on habit persistence and the equity premium puzzle is disconcerting since a major criticism of the literature on cycles and overlapping generations models has been the empirical implausibility of downward sloping savings functions.

\(^5\)Huang and Madden (1995) show that there can be cycles in an overlapping generations model with production even without a negative interest elasticity of savings as long as labor demand is sufficiently inelastic.
The rest of the paper is organized as follows: in section 2 we present the model, derive the savings function and examine its properties; the dynamic implications of the model are studied in section 3 while section 4 concludes.

2. The Model and Savings Behavior

We consider a pure exchange overlapping generations economy where agents live for two periods. There is no population growth. Agents have non-negative stationary endowments, \( w_1, w_2 \), in the two periods of their lives and maximize the present discounted value of lifetime utility given by

\[
V = u(c_1) + \beta u(c_2 - \gamma c_1)
\]

(1)

where \( u \) is the felicity function which is assumed to be strictly increasing, twice differentiable and concave while \( \beta \) is the discount factor. \( \gamma \) is the habit persistence parameter which measures the intensity of habits and is assumed to be positive but less than unity. The special case where \( \gamma = 0 \) corresponds to the standard case with no habit persistence. It is instructive to draw a few indifference curves of this utility function and the utility function without habit persistence, i.e., with \( \gamma = 0 \) (see figure 1). The slope of the indifference curve is

\[
\frac{dc_1}{dc_2} = - \frac{u'(c_1) - \gamma \beta u'(c_2 - \gamma c_1)}{\beta u'(c_2 - \gamma c_1)}
\]

(2)

(Place Figure 1 about here)
Note that for the indifference curves to be downward sloping, which is the economically meaningful area, we need to restrict the parameters of the economy such that \( \frac{u'(c_1)}{u'(c_2 - \gamma c_1)} > \gamma \beta \). It is easy to check that the indifference curves are strictly convex. From (2) one can see that for any given \( c_1 \) and \( c_2 \), the larger the \( \gamma \) the flatter the indifference curves. Thus, increasing habit persistence makes the indifference curves flatter. The "effective" consumption in the second period, \( c_2 - \gamma c_1 \), is positive above the line \( c_2 = \gamma c_1 \) in Figure 1. The indifference curves have a slope of zero along the upward sloping curve \( u'(c_1) = \gamma \beta u'(c_2 - \gamma c_1) \). If the consumer's second period endowment is zero then savings are strictly positive for any non-negative interest factor. However, the endowment point may lie above or below the line \( c_2 = \gamma c_1 \).

The periodic budget constraints faced by the typical agent during the two periods of her life are \( c_1 = w_1 - s \) and \( c_2 = Rs + w_2 \) where \( c_1 \) and \( c_2 \) denote first and second period consumption, \( s \) denotes savings and \( R \) denotes the interest factor. The savings function for the representative young agent is given by

\[
\hat{s} = s(R) = \arg\max_s u(w_1 - s) + \beta u((R + \gamma)s + w_2 - \gamma w_1) \tag{3}
\]

Note that in (3) we have substituted the periodic budget constraints into the utility function. The first order condition for the problem is given by

\[
u'(w_1 - s) = (R + \gamma)\beta u'((R + \gamma)s + w_2 - \gamma w_1) \tag{4}
\]

It is easy to check that the second order condition for utility maximization is satisfied. Differentiating the first-order condition, equation (4), with respect to \( s \), \( R \) and \( \gamma \) we get
\[
\frac{\bar{\delta}}{\partial \gamma} = \frac{\beta u'(\hat{c}_2) + (R + \gamma)(s - w_1)\beta u''(\hat{c}_2)}{-D} \tag{5}
\]

\[
\frac{\bar{\delta}}{\bar{\hat{c}}_R} = \frac{\beta u'(\hat{c}_2) \left[ 1 - \frac{(R + \gamma)s}{(R + \gamma)s + w_2 - \gamma w_1} \right] \sigma(\hat{c}_2)}{-D} \tag{6}
\]

where \(\hat{c}_2 = c_2 - \gamma c_1\). Further, \(\sigma(\hat{c}_2) = -\frac{\hat{c}_2 u''(\hat{c}_2)}{u'(\hat{c}_2)}\) which is the coefficient of relative risk aversion while \(D = u''(c_1) + (R + \gamma)^2 \beta u''(\hat{c}_2) < 0\).

The first result to note is that from equation (5) above, \(\frac{\bar{\delta}}{\partial \gamma} > 0\), i.e., an increase in habit persistence increases savings for every given interest factor and endowment profile. This follows from the fact that \(w_1 \geq s\). Thus, savings of agents with habit persistence are greater than those without habit persistence. The result is fairly obvious because current consumption puts a floor for future consumption, and thus increases the incentive to save. To get further intuition on this result consider an agent with habit persistence preferences whose effective second period income is equal to \(w_2\), i.e., she is compensated for the "loss" of income amounting to \(\gamma w_1\). Then, the decision problem that she faces is the same as that of the consumer without habit persistence except that she faces the effective interest factor \(R + \gamma\) which increases her savings.

Now consider the responsiveness of savings to the rate of interest as given by equation (8). One can see from (6) that the responsiveness of savings depend on the relative risk aversion parameter, \(\sigma\). First, consider the case of no habit persistence, i.e., \(\gamma = 0\). Under this case equation (6) reduces to
\[
\frac{\bar{c}}{\partial R} = \beta u(\hat{c}_2) \left[ 1 - \frac{R_s}{R_s + w_2} \sigma(\hat{c}_2) \right] \]

where \( \hat{c}_2 \) and \( c_2 \) are now identical. In this event, a necessary but not sufficient condition for the savings function to be decreasing in the interest rate is \( \sigma \) strictly greater than unity. Comparing the right hand sides of (6) and (7) it is easy to see that when \( \gamma > 0 \) savings can be a decreasing function of the interest rate even though \( \sigma \) is less than unity as long as endowments are such that \( w_2 < \gamma w_1 \). Thus, the savings function could be downward sloping for a consumer with habit persistence while it is upward sloping for a consumer without habit persistence who is otherwise identical. Figure 2 illustrates this possibility through the use of offer curves.

(Insert Figure 2 about here)

It is easiest to make the point through an example. Suppose the agent’s preferences belong to the family of Constant Relative Risk Aversion (CRRA) functions. In particular, let \( u(c) = \frac{c^{1-\sigma}}{1-\sigma} \) where \( \sigma \) is positive. Under this assumption, \( \sigma(c) = \sigma \) is a constant. It is quite easy, in this case, to choose parameter values such that \( \frac{\bar{c}}{\partial R} < 0 \) even though \( \sigma < 1 \).

The reason behind this result is very similar to the reason behind the relative success of models with habit persistence preferences in explaining the equity premium puzzle. Within the context of our example, for endowments such that \( w_2 < \gamma w_1 \), habit persistence increases risk
aversion without increasing \( \sigma \) which is the conventional measure of relative risk aversion. This occurs due to the fact that relative risk aversion is measured over consumption purchases, \( c \), while \( \sigma \) is measured over actual consumption services, \( \hat{c} \). Under habit persistence consumption services are smaller than consumption purchases which causes \( \sigma \) to be smaller than the relative risk aversion measure \( \frac{-cu'(\hat{c})}{u'(\hat{c})} \). It is this feature of habit persistence preferences that enable the savings function to slope downward for lower values of \( \sigma \) in our model as well as enable the asset pricing literature to account for the size of the equity premium using lower values of \( \sigma \).

3. Dynamic Equilibria

In order to analyze the dynamics of the model, we introduce an outside asset into the economy. In particular, we assume that there is a government that borrows from and lends to the public. Government liabilities in period \( t \) are denoted by \( b_t \), and the real deficit by \( d_t \). The government's budget constraint for period \( t \) is given by

\[
d_t + R_{t-1}b_{t-1} = b_t
\]

For the moment we assume that \( d_t = 0 \) for all \( t \). Asset market equilibrium requires that \( b_t = s_t \) for all \( t \). Substituting the savings function into the government budget constraint without deficits yields the equation of motion for the economy which is given by

\[
s_t(R_{t+1}) = R_t s(R_t)
\]

\[6\text{Deficits equal government expenditures minus taxes. In this paper we do not explicitly consider expenditures and taxes individually and, instead, focus on stationary deficits.}\]
In general, the difference equation given by (9) has two stationary solutions: (a) 
\[ s(R) = 0 \text{ and } R = s^{-1}(0) \quad \forall t; \] 
and (b) \[ s(1) \neq 0 \] \text{ and } \[ R = 1 \quad \forall t. \] Solution (a) is the inside-money solution while (b) is the outside-money solution.

Equation (9) can be analyzed in different ways. The geometric technique of reflected generational offer curves developed by Cass, Okuno and Zilcha (1979) is perhaps the most commonly used technique. Following their approach, we depict the offer curves for the case of positive outside-money in figure 3. The offer curve for the economy with habit persistence lies to the right of the offer curve for the economy without habit persistence since, as we showed above, savings are greater in the economy with habit persistence for any given rate of interest. Note that the interest rate is given by the slope of the ray from the origin.

(Insert Figure 3 about here)

The greater savings induced by habit persistence has non-trivial implications for the role of money in this economy. In particular, it implies that the presence of habit persistence preferences can potentially transform an otherwise classical economy into a Samuelsonian economy. Note that an economy is classical if savings in the stationary state are non-positive \( (s(1) \leq 0) \) and Samuelsonian if they are positive \( (s(1) > 0) \). Thus, habit persistence creates a role for the

---

7Assuming no population growth an economy is classical if the marginal rate of substitution of the agents at the endowment point is greater than one while it is Samuelson if the marginal rate of substitution at the endowment point is less than one.

8Brock and Scheinkman (1980) use a different geometric technique that directly considers the equilibrium sequence of interest factors, i.e. a sequence of R's which fulfill equation (12).
provision of government currency when it is otherwise absent. The key determinant of this result is the induced increase in desired savings. As we noted in footnote 4, our result is similar to the result obtained by Azariadis and Smith (1993). In their model the phenomenon of credit rationing due to an adverse selection problem causes an increase in desired savings while in our model the increase in savings is induced by habit persistence preferences.

(Insert Figure 4 about here)

As is well known, the dynamic stability properties of the two economies are very different. The classical economy has a continuum of non-stationary equilibria which converge to the monetary steady state. The Samuelson economy, on the other hand, has a continuum of non-stationary equilibria which converge to the autarkic stationary state with inside money, i.e., $s = 0$.

As an example consider the case where $u(c) = \ln(c)$. The savings function is given by

$$\hat{s} = \frac{\beta}{1 + \beta} w_1 - \frac{w_2 - \gamma w_1}{(R + \gamma)(1 + \beta)}$$

In the case where there is no habit persistence, i.e., $\gamma = 0$, (10) reduces to

$$s^* = \frac{\beta}{1 + \beta} w_1 - \frac{1}{R(1 + \beta)} w_2$$

(11)

It is easy to see that $\hat{s} > s^*$. Solving for an equilibrium without government, i.e., solving for the interest rate in such a way that savings are zero we obtain $R^* = \frac{w_2}{\beta w_1}$ and $\hat{R} = \frac{w_2}{\beta w_1} - \gamma \frac{1 + \beta}{\beta}$ which implies that $R^* > \hat{R}$. Assume that $w_2/\beta w_1 > 1$ so that the economy without habit persistence is
classical in nature. To solve explicitly for the reflected generational offer curves we invert the savings functions, substitute for \( R_i \) in (9) and obtain

\[
\begin{align*}
   s_i^* &= \frac{w_2 s_i^*}{\beta w_i - (1 + \beta)s_i^*} \\
   \hat{s}_i &= \frac{(w_2 - \gamma w_i)\hat{s}_i}{\beta w_i - (1 + \beta)\hat{s}_i} - \gamma \hat{s}_i
\end{align*}
\]  

(12) 

(13)

where, as before, starred variables pertain to the case of no habit persistence while the variables with hats pertain to the habit persistence case. The steady state savings for the two cases are given by

\[
s^* = \frac{\beta w_i - w_2}{1 + \beta} < 0
\]

(14)

\[
\hat{s} = \frac{\beta w_i - w_2}{1 + \beta} + \frac{\gamma (w_2 + w_i)}{(1 + \gamma)(1 + \beta)}
\]

(15)

By assumption \( s^* \) is negative.\(^9\) It is easy to see that \( \hat{s} \) can be positive for a large enough \( \gamma \) since the second term on the right hand side of (15) is increasing in \( \gamma \). The intensity of habit persistence (the size of \( \gamma \)) that is needed to transform the classical economy into the Samuelsonian one depends on the size of \( \beta w_i - w_2 \).

We now turn to the implications of habit persistence for the maximum feasible size of the government deficit in steady state. Suppose the government runs stationary deficits every period. The government budget constraint is given by equation (8) where \( d_i \) is replaced by \( d \). The maximum stationary deficit sustainable in the economy without habit persistence can be obtained

---

\(^9\)This the case used by Farmer (1986) to generate cycles.
by choosing \( R \) to maximize \((1-R)s(R)\). This function has two solutions \( R = 1 \) and \( R = R^* \). Note that we need \( R^* < 1 \) for the Samuelson case which is the case of interest over here. Given that \( \hat{s} > s^* \), we can depict this case through a diagram as shown in figure 5. It is easy to see that it is possible to sustain larger stationary deficits in the economy with habit persistence.

(Insert Figure 5 about here)

It is well known that in order to generate cycles in this model the reflected generational offer curves must bend backwards. As shown in section II above, the savings function under habit persistence can be a decreasing function of the rate of interest, which is the condition needed for backward bending offer curves. More importantly, recall that relative to the standard case of no habit persistence, the savings function under habit persistence can be downward sloping for lower values of the coefficient of relative risk aversion, \( \sigma(c) \). In order to illustrate the possibility of cycles in this model which arise solely due to the presence of habit persistence preferences, we consider the case of logarithmic preferences. We maintain the assumption that \( w_2 - \gamma w_1 < 0 \). Differentiating equation (9) we get the slope of the reflected generational offer curve:

\[
\frac{ds_{t+1}}{ds_t} = R \left[ 1 + \frac{1}{\varepsilon} \right]
\]

(16)

where \( \varepsilon \) is the gross interest elasticity of savings, i.e., \( \varepsilon = \frac{ds}{dR} \frac{R}{s} \). As is well known, for cycles to emerge in this model, the slope of the offer curve at the steady state point must not only be negative
but also less than one in absolute value. From (16) it is easy to see that for cycles to emerge in this model we must have $\varepsilon < -1/2$. For logarithmic preferences, $\varepsilon$ is given by

$$\varepsilon = \frac{1}{(1+\gamma)(1+\gamma)(\beta w_1 - (w_2 - \gamma w_1))} \left( w_2 - \gamma w_1 \right)$$

(17)

In the case where $w_2 = 0$, (17) reduces to

$$\varepsilon = \frac{1}{(1+\gamma)(\beta + \gamma)} \frac{-\gamma}{(1+\gamma)(\beta + \gamma)}$$

(18)

For a small enough $\beta$ the right hand side of (18) can easily be less than -1/2 for appropriate choices of $\gamma$. For example, at the limit with $\beta = 0$ and for all $\gamma$ such that $0 < \gamma < 1$, $\varepsilon < -1/2$.\footnote{With positive discounting it is easy to check that the model would generate cycles for $\beta = 1/10$ and $\gamma = 1/2$. Note that the discount factor in the overlapping generations model pertains to the second period of life which comprises of several years. Thus, it should be a much smaller number than the conventional measures that are used in infinite horizon models.} Note that we assumed at the outset that $\gamma$ is bounded between zero and one. It is worth noting that when habit persistence is absent and $w_2 = 0$, savings under logarithmic preferences are independent of the interest rate and, hence, cannot be downward sloping. Thus, in terms of our example, cycles cannot arise in the absence of habit persistence.

In closing we should also note that besides generating cycles, the backward bending offer curve is also necessary for sunspot equilibria. The connection between cycles and sunspots was made precise by Azariadis and Guesnerie (1986) who showed that a necessary and sufficient condition for the existence of sunspot equilibria is the existence of an endogenous cycle. While we do not explicitly investigate the issue of sunspots in this paper, we believe that their results should be applicable to the model developed here.
4. Conclusions

This paper has studied habit persistence preferences in the context of a simple overlapping generations model under pure exchange. In particular, we analyzed the implications of habit persistence for the savings behavior of private agents, sustainable levels of government budget deficits, and the equilibrium dynamics of the model.

The paper has shown that habit persistence unambiguously increases savings. The habit persistence induced increase in savings has a number of consequences. First, it could convert an otherwise “classical” economy into a “Samuelsonian” economy. A related result is that habit persistence preferences could create a role for the provision of government currency when no such role exists in the standard case of no habit persistence. Second, relative to the case of standard preferences, the increased savings under habit persistence make it feasible for the government to float larger stationary deficits.

The paper has shown that habit persistence preferences could have dramatic effects on the stability properties of the stationary state of the model. In particular, it could convert an economy with a stable monetary steady state into an economy with an unstable monetary steady state where the only stable stationary state is the generationally autarkic solution with inside money.

We have also shown that relative to the standard case of no habit persistence, the presence of habit formation induces savings to decline with the interest rate at lower values for the coefficient of relative risk aversion. Habit persistence induced downward sloping savings functions, in turn, could generate cycles and sunspot equilibria when such equilibria would otherwise be absent. Given the general criticism of the literature on cycles and sunspots for its
reliance on downward sloping savings functions, we find this result disconcerting since it is precisely this feature of habit persistence preferences that enables the asset pricing literature to account for the equity premium puzzle by using these preferences.
References


\[ u'(c_1) = \psi_\beta u'(c_2 - \psi c_1) \]

\[ c_2 = \psi c_1 \]
FIGURE 2.